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**SYMMETRICAL 2-EXTENSIONS OF THE 3-DIMENSIONAL GRID
WITH ALL CONNECTIONS OF TYPE 2**

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The investigation of symmetrical q -extensions of a d -dimensional grid Λ^d is of interest both for group theory and for graph theory. For small $d \geq 1$ and $q > 1$ (especially for $q = 2$), symmetrical q -extensions of Λ^d are of interest for molecular crystallography and some physical theories. Earlier V.I. Trofimov proved that there are only finitely many (up to equivalence) realizations of symmetrical 2-extensions of Λ^d for any positive integer d . E.A. Konovalchik and K.V. Kostousov found all, up to equivalence, realizations of symmetrical 2-extensions of the grid Λ^2 . Then, in the Part I of the study devoted to the case with $d = 3$, K.V. Kostousov found all, up to equivalence, realizations of symmetrical 2-extensions of the grid Λ^3 for which only the trivial automorphism preserves all blocks of σ . Among other realizations of symmetrical 2-extensions of the grid Λ^3 the realizations, in which every vertex is adjacent with only one vertex in each adjacent block, compound an important subclass. In this work we find all of them, up to equivalence.

Keywords: symmetrical extension of a graph, d -dimensional grid.

Е. А. Коновальчик, К. В. Костосов. Симметрические 2-расширения 3-мерной решетки со всеми связями типа 2.

Исследование симметрических q -расширений d -мерной решетки Λ^d представляет интерес для теории групп и теории графов. Для небольших $d \geq 1$ и $q > 1$ (особенно для $q = 2$) исследование симметрических q -расширений решетки Λ^d актуально также в связи с молекулярной кристаллографией и некоторыми физическими теориями. Ранее в работе В.И. Трофимова доказана конечность (с точностью до эквивалентности) числа реализаций симметрических 2-расширений решетки Λ^d для произвольного целого положительного d . Е.А. Коновальчик and К.В. Костосов нашли все, с точностью до эквивалентности, реализации симметрических 2-расширений решетки Λ^2 . Затем, в части I работы, посвященной случаю $d = 3$, К.В. Костосовым были найдены все, с точностью до эквивалентности, реализации симметрических 2-расширений решетки Λ^3 , для которых только единичный автоморфизм оставляет все блоки на месте. Среди остальных реализаций симметричных 2-расширений решетки Λ^3 важный подкласс составляют реализации в которых каждая вершина связана только с одной вершиной в каждом смежном блоке. В настоящей работе мы находим их все, с точностью до эквивалентности.

Ключевые слова: симметрическое расширение графа, d -мерная решетка.

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Introduction

Recall that, for a positive integer d , the d -dimensional grid Λ^d is the graph whose vertices are integer tuples (a_1, \dots, a_d) and two vertices (a'_1, \dots, a'_d) and (a''_1, \dots, a''_d) are adjacent if and only if $|a'_1 - a''_1| + \dots + |a'_d - a''_d| = 1$. According to [1], for a finite graph Δ , define a connected graph Γ to be a *symmetrical extension of Λ^d by Δ* if there exists a vertex-transitive group G of automorphisms of Γ and an imprimitivity system σ of G on $V(\Gamma)$ such that subgraphs of Γ generated by blocks of σ are isomorphic to Δ and there exists an isomorphism φ of Γ/σ (i.e., of the factor-graph of Γ by the partition σ) onto Λ^d . A tuple $(\Gamma, G, \sigma, \varphi)$ with specified components is called a *realization of the symmetrical extension Γ of the grid Λ^d by the graph Δ* . For a positive integer q , a graph Γ is called a *symmetrical q -extension of the grid Λ^d* , if Γ is a symmetrical extension of the grid Λ^d by some graph Δ such that $|V(\Delta)| = q$. In this situation the tuple $(\Gamma, G, \sigma, \varphi)$ with specified components is called a *realization of the symmetrical q -extension Γ of the grid Λ^d* , and we say that Γ is the graph of this realization. Along with purely mathematical interest, symmetrical q -extensions of the grid Λ^d for small $d \geq 1$ and $q > 1$ are interesting for crystallography and some physical theories (see [2]). For

crystallography, symmetrical 2-extensions of grids Λ^d are of the most interest. They naturally arise when considering “molecular” crystals whose “molecules” consist of two “atoms” or, more generally, have a distinguished axis.

It is natural to consider realizations of symmetrical q -extensions of the grid Λ^d up to equivalence defined as follows (see [2]). We call two such realizations $R_1 = (\Gamma_1, G_1, \sigma_1, \varphi_1)$ and $R_2 = (\Gamma_2, G_2, \sigma_2, \varphi_2)$ *equivalent* if there exists an isomorphism of the graph Γ_1 to the graph Γ_2 which maps σ_1 onto σ_2 . The realization $(\Gamma, G, \sigma, \varphi)$ of the symmetrical q -extension of the grid Λ^d will be called *maximal* if $G = \text{Aut}_\sigma(\Gamma)$ is the group of all automorphisms of the graph Γ which preserve the partition σ . It is clear that each realization of the symmetrical q -extension of the grid Λ^d has an equivalent maximal realization (unique up to equivalence). So to determine a realization $(\Gamma, G, \sigma, \varphi)$ of the symmetrical q -extension of the grid Λ^d up to equivalence it is sufficient to determine Γ, σ and φ . V.I. Trofimov proved that, up to equivalence, for an arbitrary positive integer d , there is only a finite number of realizations of symmetrical 2-extensions of the d -dimensional grid (see [3, Theorem 2]). An algorithm for constructing these extensions is also proposed in [3]. Earlier in [4] and [5] we described all, up to equivalence, realizations of symmetrical 2-extensions of Λ^2 .

In the present study, we carry out a computer realization of the algorithm, proposed in [3], for $d = 3$. The study for $d = 3$ was started in [6]. In the paper [6] (we call it Part I of the study), there were listed all, up to equivalence, realizations $(\Gamma, G, \sigma, \varphi)$ of symmetrical 2-extensions of the grid Λ^3 such that only the trivial automorphism of the graph Γ fixes all blocks of the imprimitivity system σ . By Proposition 4 from [3] these realizations and the remained ones make up classes I and II, respectively, which are defined as follows.

For an arbitrary realization $(\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^d and an arbitrary pair of adjacent vertices B_1, B_2 of the graph Γ/σ , the set of edges of the graph Γ , one end of which lies in B_1 and the other in B_2 , will be called a *connection*. The following types of connections are possible: *type 1* means a single edge; *type 2* means two non-adjacent edges; *type 2_v* means two adjacent edges; *type 3* means three edges; *type 4* means complete connection (4 edges). A realization that has connections not only of types 2 and 4 is called a *realization of class I*. A realization that has connections only of types 2 and 4 (maybe only of one type) is called a *realizations of class II*.

All realizations of class II, up to equivalence, will be listed in Part II of the study. In the present paper we consider an important subclass of class II, consisting of realizations of symmetrical 2-extensions of Λ^3 with all connections of type 2.

Following [4], realizations of symmetrical extensions of the grid Λ^d by the graph K_2 (complete graph on two vertices) are called *saturated*. Respectively, the realizations of symmetrical extensions of the grid Λ^d by the complement graph of K_2 are called *non-saturated* realizations of the symmetrical 2-extensions of the grid Λ^d . If we take a saturated realization $(\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^d and after removing all edges of Γ inside all blocks σ , we get a connected graph Γ' , then $(\Gamma', G, \sigma, \varphi)$ is a non-saturated realization of symmetrical 2-extension of the grid Λ^d . In this case we call $(\Gamma, G, \sigma, \varphi)$ and $(\Gamma', G, \sigma, \varphi)$ to be *associated*. Each non-saturated realization of symmetrical 2-extension of the grid Λ^d can be obtained in such a way from the associated saturated realization of symmetrical 2-extension of the grid Λ^d , and two non-saturated realizations of symmetrical 2-extensions of the grid Λ^d are equivalent if and only if their associated saturated realizations are equivalent.

In [6], the system $\mathbf{H} = \{H_1, \dots, H_{786}\}$ of representatives of conjugate classes of vertex-transitive subgroups of the group $\text{Aut}(\Lambda^3)$ is given. For a realization $R = (\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^3 , an arbitrary element g of the group G induces a permutation on σ , which is denoted by g^σ , and, therefore, the group G induces a permutation group on σ , which is denoted by G^σ . This permutation group is a vertex-transitive group of automorphisms of the graph Γ/σ . The group $\varphi G^\sigma \varphi^{-1}$ is conjugate in $\text{Aut}(\Lambda^3)$ with some group $H \in \mathbf{H}$.

According to [5], for any saturated realization $R = (\Gamma, G, \sigma, \varphi)$ of class II we define a subgraph $\Sigma = \Sigma(R)$ of the grid Λ^3 as follows. The vertex set of Σ coincides with $V(\Lambda^3)$, and two vertices $v_1, v_2 \in V(\Sigma)$ are adjacent in Σ if and only if there is a connection of type 2 between the blocks $\varphi^{-1}(v_1)$

and $\varphi^{-1}(v_2)$ of Σ . We will say that Σ is the *subgraph of connections of type 2 of R* . It is clear that the graph Σ admits the group $\varphi G^\sigma \varphi^{-1}$, which is conjugate in $\text{Aut}(\Lambda^3)$ with some group $H \in \mathbf{H}$.

According to [5], a subgraph Σ of the grid Λ^3 will be called *admissible*, if $V(\Sigma) = V(\Lambda^3)$ and Σ is admissible with respect to some vertex-transitive subgroup of the group $\text{Aut}(\Lambda^3)$. Two admissible subgraphs Σ_1 and Σ_2 of the grid Λ^3 we will call *equivalent*, if one of them is mapped onto the other under the action of some automorphism from $\text{Aut}(\Lambda^3)$. It is clear that subgraphs of connections of type 2 of equivalent saturated realizations of class II are equivalent too. In Part II it will be shown that up to equivalence there are 373 admissible subgraphs Σ of the grid Λ^3 .

Among all admissible subgraphs Σ of the grid Λ^3 , in the present paper we consider the case when Σ coincides with Λ^3 , i.e. to the case of realizations with all connections of type 2. This case seems the most complicated and interesting since each connection of type 2 can potentially be realized in two ways, while each connection of type 4 can be realized uniquely. The present paper is devoted to this case (the other 372 admissible subgraphs will be considered in the Part II of the work). We found¹ that in this case up to equivalence there are 480 saturated and 479 non-saturated realizations (see Sect. 3). They are listed in Sect. 4 and 5. For some subcases of this case, besides the computer implementation of the approach from [3] (see Sect. 2), a computer-free combinatorial approach is possible (see Remark 1 in Sect. 1). In this approach symmetrical 2-extensions of Λ^3 are built on the base of symmetrical 2-extensions of Λ^2 .

1. Layer Decomposable Saturated Realizations of Symmetrical 2-Extensions of the Grid Λ^3

Let $R = (\Gamma, G, \sigma, \varphi)$ be a realization of a saturated symmetrical 2-extension of Λ^3 . For $i, j, k \in \mathbb{Z}$, by $B_{i,j,k}$ we denote the block $\varphi^{-1}((i, j, k))$ of the partition σ . For $k \in \mathbb{Z}$, consider the subgraph Γ' of Γ , generated by the set of vertices $V = \bigcup_{i,j \in \mathbb{Z}} B_{i,j,k}$. The triple $(\Gamma', \sigma|_V, \varphi|_V)$, where $\sigma|_V = \{\varphi^{-1}((i, j, k)) : i, j \in \mathbb{Z}\}$, will be called a *cutting of R by fixing third coordinate*. In the same manner for $j \in \mathbb{Z}$ and subgraph Γ' generated by the set $V = \bigcup_{i,k \in \mathbb{Z}} B_{i,j,k}$, the triple $(\Gamma', \sigma|_V, \varphi|_V)$ will be called a *cutting of R by fixing second coordinate*. Finally for $i \in \mathbb{Z}$ and the subgraph Γ' generated by the set $V = \bigcup_{j,k \in \mathbb{Z}} B_{i,j,k}$, the triple $(\Gamma', \sigma|_V, \varphi|_V)$ will be called a *cutting of R by fixing first coordinate*. Two cuttings of the realization R by fixing the same coordinate will be called *parallel*.

Note that if for some cutting $(\Gamma', \sigma', \varphi')$ of a symmetrical 2-extension of Λ^3 the group G' of all automorphisms from $\text{Aut}(\Gamma')$, preserving blocks σ' , acts transitively on $V(\Gamma')$, then $(\Gamma', G', \sigma', \varphi')$ is a realization of a saturated symmetrical 2-extension of the grid Λ^2 , and we say that the cutting $(\Gamma', \sigma', \varphi')$ *corresponds* to a realization of a symmetrical 2-extension of the grid Λ^2 . Two cuttings $R_1 = (\Gamma_1, \sigma_1, \varphi_1)$ and $R_2 = (\Gamma_2, \sigma_2, \varphi_2)$ are called *equivalent*, if there exists an isomorphism of the graph Γ_1 onto Γ_2 which maps σ_1 onto σ_2 .

We say that a realization of a saturated symmetrical 2-extension of the grid Λ^3 is *layer decomposable*, if each its cutting corresponds to a saturated realization of a symmetrical 2-extension of the grid Λ^2 . In [5, Table 3] all, up to equivalence, saturated realizations of symmetrical 2-extensions of Λ^2 of class II are listed: there are 38 realizations which we denote now by $R_1^2, R_2^2, \dots, R_{38}^2$ respectively. First 8 of these realizations contain only connections of type 2. Therefore each cutting of a layer decomposable realization of a symmetrical 2-extension of Λ^3 containing only connections of type 2 correspond to a saturated realization of symmetrical 2-extension of Λ^2 which is equivalent to one of $R_1^2, R_2^2, \dots, R_8^2$.

Proposition 1. *If some cutting C of a saturated realization R of a symmetrical 2-extension of the grid Λ^3 corresponds to a saturated realization of a symmetrical 2-extension of the grid Λ^2 , then all parallel to C cuttings of R correspond to a saturated realizations of symmetrical 2-extensions of the grid Λ^2 , and they are equivalent to C .*

¹Our computations were performed on the Uran supercomputer at the IMM UB RAS.

Доказательство. Let $R = (\Gamma, G, \sigma, \varphi)$ be a saturated realization of a symmetrical 2-extension of Λ^3 . Let $(\Gamma_1, \sigma_1, \varphi_1)$, $(\Gamma_2, \sigma_2, \varphi_2)$ and $(\Gamma_3, \sigma_3, \varphi_3)$ be cuttings of R by fixing 1st, 2nd and 3rd coordinates respectively and such that $\varphi^{-1}((0, 0, 0)) \in \sigma_1 \cap \sigma_2 \cap \sigma_3$. Let $(\Gamma_4, \sigma_4, \varphi_4)$ be a cutting of R by fixing 1st coordinate such that $\varphi^{-1}((i, 0, 0)) \in \sigma_4$ for some integer i . Since $\varphi^{-1}((i, 0, 0)) \in \sigma_2 \cap \sigma_3$, the symmetricity of R implies that there is a bijection from $\{(\Gamma_1, \sigma_1, \varphi_1), (\Gamma_2, \sigma_2, \varphi_2), (\Gamma_3, \sigma_3, \varphi_3)\}$ onto $\{(\Gamma_2, \sigma_2, \varphi_2), (\Gamma_3, \sigma_3, \varphi_3), (\Gamma_4, \sigma_4, \varphi_4)\}$, such that each cutting is mapped to an equivalent one. Therefore cuttings $(\Gamma_1, \sigma_1, \varphi_1)$ and $(\Gamma_4, \sigma_4, \varphi_4)$ are equivalent to each other.

In the same manner it is easy to prove that any two parallel cuttings of R are equivalent to each other. \square

Proposition 1 implies that the condition in the definition of a layer decomposable saturated realization can be made weaker, and we get the following layer decomposability criterion.

Proposition 2. *A saturated realization of asymmetrical 2-extension of the grid of a symmetrical 2-extension of the grid Λ^3 is layer decomposable if and only if some three pairwise non-parallel cuttings correspond to saturated realizations of symmetrical 2-extensions of Λ^2 .*

We will say, that a saturated realization of a symmetrical 2-extension of the grid Λ^3 of class II has a descriptor $[n_1, n_2, n_3]$, where n_1, n_2, n_3 are integers such that $1 \leq n_1 \leq n_2 \leq n_3 \leq 38$, if all cuttings of R by fixing one of coordinates are equivalent to $R_{n_1}^2$, and all cuttings of R by fixing another coordinate are equivalent to $R_{n_2}^2$, and all cuttings of R by fixing the remained coordinate are equivalent to $R_{n_3}^2$. We expand this definition of the descriptor to the case where some cuttings of R do not correspond to saturated realizations of symmetrical 2-extensions of Λ^2 , putting $n_3 = -$, or $n_2 = n_3 = -$, or $n_1 = n_2 = n_3 = -$. For example, $[3, -, -]$ is a descriptor of a saturated realization R such that all cuttings of R by fixing of one of coordinates correspond to a saturated realization, which is equivalent to R_3^2 and all other cuttings of R do not correspond to saturated realizations of symmetrical 2-extensions of Λ^2 .

In the terms of descriptors we can reformulate Proposition 2.

Proposition 3. *A saturated realization of a symmetrical 2-extension of the grid Λ^3 is layer decomposable if and only if its descriptor does not contain “-”.*

In Part I of our study (see [6]), we used vertex neighborhood extensions in order to arrange the extensions list. But for realizations of class II there are very few different vertex neighbourhood extensions, and so we use descriptors here to arrange the extensions list.

Remark 1. As it was said in Introduction, we use a computer implementation of the approach from [3]. However, for layer decomposable (saturated) realizations of symmetrical 2-extensions of the grid Λ^3 of class II, the authors developed a computer-free combinatorial approach which, in principle, can be used to get the list of such realizations. This approach is based on Proposition 1 and the list $R_1^2, R_2^2, \dots, R_{38}^2$ of saturated realizations of symmetrical 2-extensions of Λ^2 of class II.

2. Computer Implementation of the Approach from [3] for Realizations of Symmetrical 2-Extensions of the Grid Λ^3 with All Connections of Type 2

To construct realizations of symmetrical 2-extensions of Λ^3 with all connections of type 2, we carried out a computer implementation of the following approach proposed in [3], which can be called a coordinatization of symmetrical extensions of graphs.

Let G be a group and L be a subgroup of G . Let, in addition, \mathcal{P} be some set of two-element subsets of the form $\{L, aL\}$, $a \in G$, of the set G/L of left cosets of L in G . Then by $\Gamma_{G,L,\mathcal{P}}$ we denote the graph with vertex set G/L and edge set $\{\lambda_{G/L}(g)(P) : P \in \mathcal{P}, g \in G\}$, where $\lambda_{G/L}$ is the action of the group G on G/L by left shifts. Note that $\lambda_{G/L}(G)$ is a vertex-transitive group of automorphisms

of the graph $\Gamma_{G,L,\mathcal{P}}$. Let X be a set of elements of G , such that $\mathcal{P} = \{\{L, aL\} : a \in X\}$. Then we will also denote the graph $\Gamma_{G,L,\mathcal{P}}$ by $\Gamma_{G,L,X}$.

Now let H be a vertex-transitive group of automorphisms of the grid Λ^3 and G be a central extension of the group H by the group $\langle c \rangle$ of order 2. For any $g \in G$ by \bar{g} we denote the image of g under the natural homomorphism $G \rightarrow H$. Fix elements $g^{(1)}, \dots, g^{(6)} \in G$ such that

$$\begin{aligned} \overline{g^{(1)}}(0,0,0) &= (1,0,0), & \overline{g^{(2)}}(0,0,0) &= (-1,0,0), & \overline{g^{(3)}}(0,0,0) &= (0,1,0), \\ \overline{g^{(4)}}(0,0,0) &= (0,-1,0), & \overline{g^{(5)}}(0,0,0) &= (0,0,1), & \overline{g^{(6)}}(0,0,0) &= (0,0,-1). \end{aligned}$$

Let $K \leq G$ be a preimage of $H_{(0,0,0)}$ (the stabilizer of $(0,0,0)$ in H) under natural homomorphism $G \rightarrow H$ (in particular, $c \in K$). Let L be a subgroup of K of index 2 such that $c \notin L$ (therefore $K = L \cup cL$). Then $\sigma := \{\lambda_{G/L}(g)(\{L, cL\}) : g \in G\}$ is an imprimitivity system of the group $\lambda_{G/L}(G)$ on G/L . By φ we denote the one-to-one mapping of σ onto $V(\Lambda^3)$ such that $\varphi(\lambda_{G/L}(g)(\{L, cL\}))$ coincides with $\bar{g}((0,0,0))$. Take some tuple $(\delta_1, \dots, \delta_6)$, $\delta_1, \dots, \delta_6 \in \{0, 1\}$, and construct the set

$$X = \{c^{\delta_m} g^{(m)} : m = 1, \dots, 6\} \cup \{c\}.$$

Since $|X| = 7$, the graph $\Gamma_{G,L,X}$ will be of degree ≥ 7 . If the degree is equal to 7, then we get the saturated realization $(\Gamma_{G,L,X}, \lambda_{G/L}(G), \sigma, \varphi)$ of symmetrical 2-extension of the grid Λ^3 with all connections of type 2.

It follows from [3, Propostion 6] that, up to equivalence, every realization of symmetrical 2-extension of the grid Λ^3 with all connections of type 2 can be constructed in such a way. This approach is realized in the following algorithm.

Algorithm 1. *Generating all (up to equivalence) saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2.*

Input: *The system $\mathbf{H} = \{H_1, \dots, H_{786}\}$ of representatives of conjugate classes of vertex-transitive subgroups of the group $\text{Aut}(\Lambda^3)$.*

Output: *A list of saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2, which, up to equivalence, exhaust such realizations.*

Description:

1. Look over all vertex-transitive subgroups $H \in \mathbf{H}$ (in fact this looking over can be reduced, see Remark 2 bellow), and for each such H do the following steps.

2. Construct a polycyclic presentation of the group H given in [6] in the matrix form (see [7]; to this end we use the procedure `IsomorphismPcpGroup(H)` from GAP-package `Cryst` [8; 9]). In the obtained presentation with the set of generators g_1, \dots, g_s it is known how these generators act on Λ^3 . Besides that this set of generators is arranged such that g_{s-2}, g_{s-1}, g_s is a translation basis of the crystallographic group H . Let $C \subset \Lambda^3$ be a fundamental cell, corresponding to this basis. Next we look over all products of g_1, g_2, \dots, g_{s-3} (first of length 1, then of length 2, etc.), each product we multiply by $g_{s-2}^i g_{s-1}^j g_s^k$ for appropriate integers i, j, k such that the obtained product p maps $(0,0,0)$ to a vertex from C . We continue up to find products $p_1, p_2, \dots, p_{|C|}$ such that $\{p_i(0,0,0) : i \in \{1, \dots, |C|\}\} = C$. Further, using this products, we construct products $p^{(1)}, \dots, p^{(6)} \in H$ of g_1, g_2, \dots, g_s such that

$$\begin{aligned} p^{(1)}(0,0,0) &= (1,0,0), & p^{(2)}(0,0,0) &= (-1,0,0), & p^{(3)}(0,0,0) &= (0,1,0), \\ p^{(4)}(0,0,0) &= (0,-1,0), & p^{(5)}(0,0,0) &= (0,0,1), & p^{(6)}(0,0,0) &= (0,0,-1). \end{aligned}$$

3. Look over all central extensions of the group H by the group $\langle c \rangle$ of order 2. To find all such extensions we take generators g_1, \dots, g_s and relations $W_1(g_1, \dots, g_s), \dots, W_r(g_1, \dots, g_s)$ of polycyclic presentation of H , constructed on step 2. We add an element c to the set of generators and extend the set of relations in 2^r ways, as follws. We add the relation c^2 , the relations $g_i c g_i^{-1} c$ for $i \in \{1, \dots, s\}$, and for each list of numbers $\varepsilon_1, \dots, \varepsilon_r \in \{0, 1\}$ the relations $W_1(g_1, \dots, g_s) c^{\varepsilon_1}, \dots, W_r(g_1, \dots, g_s) c^{\varepsilon_r}$

(see the proof of Theorem 1 from [3]). Then we use the procedure IsConfluent from GAP-package Polycyclic to choose presentations where $c \neq 1$.

4. For each extension G of H by $\langle c \rangle$ constructed on step 3, fix elements $g^{(1)}, \dots, g^{(6)} \in G$ which are obtained from products $p^{(1)}, \dots, p^{(6)}$ by replacing elements g_1, \dots, g_s of H by corresponding elements of G . Let K be a preimage of the group $H_{(0,0,0)}$ in G under the natural homomorphism $G \rightarrow H = G/\langle c \rangle$. Look over all subgroups L of index 2 of the group K such that $c \notin L$. Look over all tuples $(\delta_1, \dots, \delta_6)$, where $\delta_j \in \{0, 1\}$ for $j = 1, \dots, 6$. Construct subsets $X = \{c^{\delta_m} g^{(m)} : m = 1, \dots, 6\} \cup \{c\}$. If the graph $\Gamma_{G,L,X}$ is of degree 7, make up the realization $(\Gamma_{G,L,X}, \lambda_{G/L}(G), \sigma, \varphi)$, where $\sigma = \{(l, m, n, 1), (l, m, n, 2) : l, m, n \in \mathbb{Z}\}$ and $\varphi((l, m, n, s)) = \{(l, m, n, 1), (l, m, n, 2)\}$ for $l, m, n \in \mathbb{Z}, s \in \{1, 2\}$. Put this realization into the output list. \square

To compare generated realizations for equivalence we used Algorithm 2, described in [6] and used there for realizations of class I.

Remark 2. In Algorithm 1 instead of looking over all $H \in \mathbf{H}$, it is sufficient to look over only minimal vertex-transitive subgroups, i.e. $H_i \in \mathbf{H}$ for $i \in \{1, \dots, 63\} \cup \{217, \dots, 344\} \cup \{351, 361, 370, 374, 377, 379, 399, 408, 412, 415, 417, 422, 425, 428, 430, 432, 433, 434, 435, 437, 439, 440, 444, 447, 450, 452, 454, 455, 456, 457, 459\} \cup \{461, \dots, 531\} \cup \{666, \dots, 701\} \cup \{781, 782\}$. Indeed, let $(\Gamma, G, \sigma, \varphi)$ be a maximal realization of class II. Let G_1 be a minimal subgroup of G , acting transitively on σ and containing c (i.e. the element that swaps vertices in each block of σ). Then G_1 induces on the blocks σ a minimal vertex-transitive subgroup of $\text{Aut}(\Lambda^3)$.

3. Realizations of Symmetrical 2-Extensions of the Grid Λ^3 with All Connections of Type 2

As stated in the Introduction, symmetrical 2-extensions of Λ^3 with all connections of type 2 are of special interest among all symmetrical 2-extensions of Λ^3 of class II. Using the approach from [3] for class II described above (Sect. 2), we construct all (up to equivalence) saturated realizations of symmetrical 2-extensions of Λ^3 with all connections of type 2 (these realizations are generated via Algorithm 1). Using Proposition 3 we divide them into two lists: layer decomposable realizations (see Theorem 1) and layer non-decomposable realizations (see Theorem 2).

Theorem 1. *Up to equivalence, there are 362 layer decomposable saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2. These realizations are listed in Table 1 (see Sect. 4).*

Corollary 1. *Up to equivalence, there are 361 layer decomposable non-saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2.*

Proof. It is easy to see that for 362 realizations given in Table 1, only one of their graphs (namely the graph of $R_{1,1,1_1}$) become disconnected after removing edges inside blocks.

Theorem 2. *Up to equivalence, there are 118 layer non-decomposable saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2. These realizations are listed in Table 2 (see Sect. 5).*

Corollary 2. *Up to equivalence there are 118 layer non-decomposable non-saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2.*

Proof. It is easy to see that for 118 realizations given in Table 1, all of their graphs remain connected after removing edges inside blocks.

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4. The List of Layer Decomposable Saturated Realizations of Symmetrical 2-Extensions of the Grid Λ^3 with All Connections of Type 2

Recall (see [Introduction](#)) that to determine a realization $(\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^3 up to equivalence it is sufficient to determine Γ, σ and φ .

[Table 1](#) is structured using descriptors $[n_1, n_2, n_3]$, which are lexicographically numerated by (1), (2), \dots , (102). Namely, each “heading” line of [Table 1](#) contains a descriptor number (s) (where $s \in \{1, \dots, 102\}$) and the corresponding descriptor $[n_1, n_2, n_3]$ in the first column, and the number of saturated realizations of symmetrical 2-extensions of Λ^3 with this descriptor in the second column. Between a “heading” line and the next one all realizations coresponding to $[n_1, n_2, n_3]$ are described. In the first column the realization number n and the realization name $R_{n_1, n_2, n_3; m}$ are specified. The index m is used to number non-equivalent realizations, having the same descriptor. In the second column the edge set of the graph of the realization is given by periods p_1, p_2, p_3 and sets V_1, V_2, V_3 which are defined as follows.

For the graph Γ of a saturated realization $R = (\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^3 with all connections of type 2, we put $V(\Gamma) = \{(i, j, k, l) : i, j, k \in \mathbb{Z}, l \in \{1, 2\}\}$ with

$$\sigma = \{(i, j, k, 1), (i, j, k, 2)\} : i, j, k \in \mathbb{Z} \quad \text{and} \quad \varphi : \{(i, j, k, 1), (i, j, k, 2)\} \mapsto (i, j, k)$$

for $i, j, k \in \mathbb{Z}$.

To specify the edge set of Γ , we use the following notation:

$$E(\Gamma) = E_0(\Gamma) \cup E_1(\Gamma) \cup E_2(\Gamma) \cup E_3(\Gamma),$$

where

$$\begin{aligned} E_0(\Gamma) &= \{(i, j, k, l_1), (i, j, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}, \\ E_1(\Gamma) &\subseteq \{(i, j, k, l_1), (i+1, j, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}, \\ E_2(\Gamma) &\subseteq \{(i, j, k, l_1), (i, j+1, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}, \\ E_3(\Gamma) &\subseteq \{(i, j, k, l_1), (i, j, k+1, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}. \end{aligned}$$

To describe $E_1(\Gamma), E_2(\Gamma), E_3(\Gamma)$, it is enough to define the sets

$$\begin{aligned} W_1(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i+1, j, k, 2)\} \in E_1(\Gamma)\}, \\ W_2(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i, j+1, k, 2)\} \in E_2(\Gamma)\}, \\ W_3(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i, j, k+1, 2)\} \in E_3(\Gamma)\}. \end{aligned}$$

In fact,

$$\begin{aligned} E_1(\Gamma) &= \{(i, j, k, 1), (i+1, j, k, 2)\}, \{(i, j, k, 2), (i+1, j, k, 1)\} : (i, j, k) \in W_1(\Gamma) \\ &\cup \{(i, j, k, 1), (i+1, j, k, 1)\}, \{(i, j, k, 2), (i+1, j, k, 2)\} : (i, j, k) \notin W_1(\Gamma), \\ E_2(\Gamma) &= \{(i, j, k, 1), (i, j+1, k, 2)\}, \{(i, j, k, 2), (i, j+1, k, 1)\} : (i, j, k) \in W_2(\Gamma) \\ &\cup \{(i, j, k, 1), (i, j+1, k, 1)\}, \{(i, j, k, 2), (i, j+1, k, 2)\} : (i, j, k) \notin W_2(\Gamma), \\ E_3(\Gamma) &= \{(i, j, k, 1), (i, j, k+1, 2)\}, \{(i, j, k, 2), (i, j, k+1, 1)\} : (i, j, k) \in W_3(\Gamma) \\ &\cup \{(i, j, k, 1), (i, j, k+1, 1)\}, \{(i, j, k, 2), (i, j, k+1, 2)\} : (i, j, k) \notin W_3(\Gamma). \end{aligned}$$

The sets $W_1(\Gamma), W_2(\Gamma)$ and $W_3(\Gamma)$ are periodic with some positive periods p_1, p_2 and p_3 (for the concept of $[p_1, p_2, p_3]$ -periodicity of a realization see [\[2\]](#)). It means, that

$$\begin{aligned} W_1(\Gamma) &= \{(i + ap_1, j + bp_2, k + cp_3) : (i, j, k) \in V_1(\Gamma), a, b, c \in \mathbb{Z}\}, \\ W_2(\Gamma) &= \{(i + ap_1, j + bp_2, k + cp_3) : (i, j, k) \in V_2(\Gamma), a, b, c \in \mathbb{Z}\}, \\ W_3(\Gamma) &= \{(i + ap_1, j + bp_2, k + cp_3) : (i, j, k) \in V_3(\Gamma), a, b, c \in \mathbb{Z}\}, \end{aligned}$$

for some subsets $V_1 = V_1(\Gamma), V_2 = V_2(\Gamma), V_3 = V_3(\Gamma)$ of the set

$$\{(i, j, k) : i \in \{0, 1, \dots, p_1 - 1\}, j \in \{0, 1, \dots, p_2 - 1\}, k \in \{0, 1, \dots, p_3 - 1\}\}.$$

Table 1. 362 layer decomposable saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2

(1) 1, 1, 1	1
1) $R_{1,1,1;1}$	$p_1 = 1, p_2 = 1, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \emptyset$
(2) 1, 1, 2	1
2) $R_{1,1,2;1}$	$p_1 = 1, p_2 = 2, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0)\}$
(3) 1, 1, 3	1
3) $R_{1,1,3;1}$	$p_1 = 1, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 3, 0)\}$
(4) 1, 1, 4	1
4) $R_{1,1,4;1}$	$p_1 = 1, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1)\}$
(5) 1, 1, 5	1
5) $R_{1,1,5;1}$	$p_1 = 1, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 7, 0)\}$
(6) 1, 1, 6	1
6) $R_{1,1,6;1}$	$p_1 = 1, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0)\}$
(7) 1, 1, 7	1
7) $R_{1,1,7;1}$	$p_1 = 1, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1)\}$
(8) 1, 1, 8	1
8) $R_{1,1,8;1}$	$p_1 = 1, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0)\}$
(9) 1, 2, 2	1
9) $R_{1,2,2;1}$	$p_1 = 2, p_2 = 2, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (1, 0, 0)\}$
(10) 1, 2, 3	2
10) $R_{1,2,3;1}$	$p_1 = 2, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 3, 0), (1, 0, 0), (1, 1, 0)\}$
11) $R_{1,2,3;2}$	$p_1 = 2, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 0, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1)\}$
(11) 1, 2, 4	2
12) $R_{1,2,4;1}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1), (1, 5, 0), (1, 5, 1), (1, 6, 1), (1, 7, 0)\}$
13) $R_{1,2,4;2}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 0, 3), (1, 1, 1), (1, 2, 0), (1, 2, 2), (1, 3, 3)\}$
(12) 1, 2, 5	2
14) $R_{1,2,5;1}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 7, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 6, 1), (1, 7, 1)\}$
15) $R_{1,2,5;2}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 2, 1), (0, 2, 3), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 0, 3), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 3, 0), (1, 3, 1), (1, 3, 2)\}$
(13) 1, 2, 6	1
16) $R_{1,2,6;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 3, 1)\}$
(14) 1, 2, 7	1
17) $R_{1,2,7;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0)\}$
(15) 1, 2, 8	1
18) $R_{1,2,8;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1)\}$
(16) 1, 3, 3	6

Table 1 (cont.)

Σ	Edges
19) $R_{1,3,3;1}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 3, 0), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 1, 0), (3, 0, 0), (3, 1, 0)\}$
20) $R_{1,3,3;2}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0)\}$
21) $R_{1,3,3;3}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (1, 1, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0)\}$
22) $R_{1,3,3;4}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (1, 1, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 0, 1), (3, 1, 1)\}$
23) $R_{1,3,3;5}$	$p_1 = 2, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (1, 0, 1)\}$
24) $R_{1,3,3;6}$	$p_1 = 2, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$
(17) [1, 3, 4]	5
25) $R_{1,3,4;1}$	$p_1 = 4, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 4, 0), (1, 4, 1), (1, 6, 0), (1, 7, 1), (2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 3, 1), (2, 5, 0), (2, 5, 1), (2, 6, 1), (2, 7, 0), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1), (3, 5, 0), (3, 5, 1), (3, 6, 1), (3, 7, 0)\}$
26) $R_{1,3,4;2}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 0, 1), (1, 0, 3), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2)\}$
27) $R_{1,3,4;3}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 2), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 3)\}$
28) $R_{1,3,4;4}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 2, 3), (1, 3, 0), (1, 3, 2), (1, 3, 3), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 0, 3), (2, 1, 3), (2, 2, 0), (2, 2, 2), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 1, 3), (3, 2, 0), (3, 2, 2), (3, 3, 1)\}$
29) $R_{1,3,4;5}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1)\}$ $V_2 = \{(0, 3, 1), (1, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
(18) [1, 3, 5]	5
30) $R_{1,3,5;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 1, 3), (1, 2, 1), (1, 2, 3), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 0, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 2, 0), (2, 2, 2), (2, 3, 0), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 2, 0), (3, 2, 2), (3, 3, 0), (3, 3, 2), (3, 3, 3)\}$
31) $R_{1,3,5;2}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 1), (1, 0, 3), (1, 1, 1), (1, 3, 3)\}$
32) $R_{1,3,5;3}$	$p_1 = 2, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 0), (1, 0, 2), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2)\}$
33) $R_{1,3,5;4}$	$p_1 = 4, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 1), (0, 7, 1), (1, 2, 0), (1, 3, 0), (1, 4, 0), (1, 4, 1), (1, 5, 0), (1, 5, 1), (1, 6, 1), (1, 7, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 1), (2, 3, 1), (2, 6, 0), (2, 7, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 1), (3, 3, 1), (3, 6, 0), (3, 7, 0)\}$
34) $R_{1,3,5;5}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1)\}$ $V_2 = \{(0, 1, 1), (1, 1, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
(19) [1, 3, 6]	4
35) $R_{1,3,6;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 1, 0), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 3, 0)\}$
36) $R_{1,3,6;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 3, 1)\}$
37) $R_{1,3,6;3}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0)\}$
38) $R_{1,3,6;4}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$

Table 1 (cont.)

Σ	Edges
	$V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1)\}$
(20) 1, 3, 7	4
39) $R_{1,3,7;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 2, 1), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 0), (3, 3, 0)\}$
40) $R_{1,3,7;2}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1)\}$
41) $R_{1,3,7;3}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$
42) $R_{1,3,7;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 1), (3, 3, 1)\}$
(21) 1, 3, 8	4
43) $R_{1,3,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1)\}$
44) $R_{1,3,8;2}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 1), (1, 1, 0), (1, 3, 0), (1, 3, 1)\}$
45) $R_{1,3,8;3}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1)\}$
46) $R_{1,3,8;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 1), (3, 3, 0)\}$
(22) 1, 4, 4	5
47) $R_{1,4,4;1}$	$p_1 = 8, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 5, 0), (1, 6, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 2, 1), (2, 4, 0), (2, 5, 1), (2, 7, 0), (2, 7, 1), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 4, 1), (3, 6, 0), (3, 6, 1), (3, 7, 1), (4, 0, 0), (4, 0, 1), (4, 2, 0), (4, 3, 1), (4, 5, 0), (4, 5, 1), (4, 6, 1), (4, 7, 0), (5, 1, 0), (5, 2, 1), (5, 4, 0), (5, 4, 1), (5, 5, 1), (5, 6, 0), (5, 7, 0), (5, 7, 1), (6, 0, 0), (6, 1, 1), (6, 3, 0), (6, 3, 1), (6, 4, 1), (6, 5, 0), (6, 6, 0), (6, 6, 1), (7, 0, 1), (7, 2, 0), (7, 2, 1), (7, 3, 1), (7, 4, 0), (7, 5, 0), (7, 5, 1), (7, 7, 0)\}$
48) $R_{1,4,4;2}$	$p_1 = 8, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (1, 4, 0), (1, 4, 1), (1, 6, 0), (1, 6, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1), (3, 4, 0), (3, 4, 1), (3, 6, 0), (3, 6, 1), (5, 0, 0), (5, 0, 1), (5, 2, 0), (5, 2, 1), (5, 4, 0), (5, 4, 1), (5, 6, 0), (5, 6, 1), (7, 0, 0), (7, 0, 1), (7, 2, 0), (7, 2, 1), (7, 4, 0), (7, 4, 1), (7, 6, 0), (7, 6, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 5, 0), (1, 6, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 2, 1), (2, 4, 0), (2, 5, 1), (2, 7, 0), (2, 7, 1), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 4, 1), (3, 6, 0), (3, 6, 1), (3, 7, 1), (4, 0, 0), (4, 0, 1), (4, 2, 0), (4, 3, 1), (4, 5, 0), (4, 5, 1), (4, 6, 1), (4, 7, 0), (5, 1, 0), (5, 2, 1), (5, 4, 0), (5, 4, 1), (5, 5, 1), (5, 6, 0), (5, 7, 0), (5, 7, 1), (6, 0, 0), (6, 1, 1), (6, 3, 0), (6, 3, 1), (6, 4, 1), (6, 5, 0), (6, 6, 0), (6, 6, 1), (7, 0, 1), (7, 2, 0), (7, 2, 1), (7, 3, 1), (7, 4, 0), (7, 5, 0), (7, 5, 1), (7, 7, 0)\}$
49) $R_{1,4,4;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2)\}$
50) $R_{1,4,4;4}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 3), (0, 2, 0), (0, 2, 2), (0, 3, 1), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 3, 0), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 2, 0), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 2), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 1, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1)\}$
51) $R_{1,4,4;5}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 1), (3, 3, 3)\}$
(23) 1, 4, 5	4
52) $R_{1,4,5;1}$	$p_1 = 8, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 7, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 2, 0), (1, 5, 1), (1, 6, 1), (1, 7, 0), (1, 7, 1), (2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (2, 4, 0), (2, 5, 0), (3, 0, 0), (3, 3, 1), (3, 4, 1), (3, 5, 0), (3, 5, 1), (3, 6, 0), (3, 6, 1), (3, 7, 0), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (4, 2, 0), (4, 3, 0), (4, 6, 1), (4, 7, 1), (5, 1, 1), (5, 2, 1), (5, 3, 0), (5, 3, 1), (5, 4, 0), (5, 4, 1), (5, 5, 0), (5, 6, 0), (6, 0, 0), (6, 1, 0), (6, 4, 1), (6, 5, 1), (6, 6, 0), (6, 6, 1), (6, 7, 0), (6, 7, 1), (7, 0, 1), (7, 1, 0), (7, 1, 1), (7, 2, 0), (7, 2, 1), (7, 3, 0), (7, 4, 0), (7, 7, 1)\}$
53) $R_{1,4,5;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 2), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 2), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$
54)	$p_1 = 4, p_2 = 4, p_3 = 4$

Table 1 (cont.)

Σ	Edges
$R_{1,4,5;3}$	$V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 2), (0, 2, 0), (0, 2, 2), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 1), (1, 3, 2), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 0), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 3), (3, 2, 3), (3, 3, 2), (3, 3, 3)\}$
55) $R_{1,4,5;4}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 2), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 2)\}$
(24) 1, 4, 6	3
56) $R_{1,4,6;1}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (3, 0, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1), (4, 0, 0), (4, 0, 1), (4, 1, 1), (4, 3, 0), (5, 1, 0), (5, 2, 0), (5, 2, 1), (5, 3, 1), (6, 0, 0), (6, 2, 1), (6, 3, 0), (6, 3, 1), (7, 0, 1), (7, 1, 0), (7, 1, 1), (7, 2, 0)\}$
57) $R_{1,4,6;2}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 3, 0), (2, 0, 1), (2, 2, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 2, 1), (4, 0, 0), (4, 0, 1), (4, 1, 1), (4, 3, 0), (5, 1, 0), (5, 2, 0), (5, 2, 1), (5, 3, 1), (6, 0, 0), (6, 1, 0), (6, 1, 1), (6, 2, 1), (7, 0, 1), (7, 2, 0), (7, 3, 0), (7, 3, 1)\}$
58) $R_{1,4,6;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 1, 3), (0, 2, 0), (0, 2, 1), (0, 2, 2), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 2, 3), (1, 3, 1), (2, 0, 1), (2, 0, 3), (2, 2, 0), (2, 2, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 2, 1), (3, 2, 2), (3, 3, 3)\}$
(25) 1, 4, 7	3
59) $R_{1,4,7;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 2, 3), (0, 3, 1), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 2), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 2), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 1), (3, 3, 2)\}$
60) $R_{1,4,7;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 2, 3), (0, 3, 1), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 2), (3, 3, 3)\}$
61) $R_{1,4,7;3}$	$p_1 = 2, p_2 = 2, p_3 = 4$ $V_1 = \{(0, 0, 1), (0, 1, 1), (1, 0, 0), (1, 1, 0)\}$ $V_2 = \{(0, 1, 1), (0, 1, 2), (1, 1, 1), (1, 1, 2)\}$ $V_3 = \{(1, 0, 2), (1, 1, 2)\}$
(26) 1, 4, 8	3
62) $R_{1,4,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 1), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 3)\}$
63) $R_{1,4,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 1)\}$
64) $R_{1,4,8;3}$	$p_1 = 2, p_2 = 2, p_3 = 4$ $V_1 = \{(0, 0, 1), (0, 1, 1), (1, 0, 0), (1, 1, 0)\}$ $V_2 = \{(0, 0, 1), (0, 0, 3), (0, 1, 2), (0, 1, 3), (1, 0, 1), (1, 0, 3), (1, 1, 2), (1, 1, 3)\}$ $V_3 = \{(1, 0, 2), (1, 1, 2)\}$
(27) 1, 5, 5	4
65) $R_{1,5,5;1}$	$p_1 = 8, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 7, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 4, 1), (1, 5, 0), (1, 6, 0), (2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (2, 4, 0), (2, 5, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 4, 0), (3, 7, 1), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (4, 2, 0), (4, 3, 0), (4, 6, 1), (4, 7, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 2, 0), (5, 5, 1), (5, 6, 1), (5, 7, 0), (5, 7, 1), (6, 0, 0), (6, 1, 0), (6, 4, 1), (6, 5, 1), (6, 6, 0), (6, 6, 1), (6, 7, 0), (6, 7, 1), (7, 0, 0), (7, 3, 1), (7, 4, 1), (7, 5, 0), (7, 5, 1), (7, 6, 0), (7, 6, 1), (7, 7, 0)\}$
66) $R_{1,5,5;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 3), (1, 2, 1), (1, 3, 1), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 3, 3), (3, 0, 1), (3, 1, 1), (3, 1, 3), (3, 2, 3)\}$
67) $R_{1,5,5;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 2), (0, 2, 0), (0, 2, 2), (0, 3, 0), (1, 0, 3), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 0), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 0), (2, 3, 1), (2, 3, 3), (3, 0, 1), (3, 1, 1), (3, 1, 2), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 0), (3, 3, 1)\}$
68) $R_{1,5,5;4}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 1), (1, 0, 3), (1, 1, 1), (1, 1, 3), (1, 2, 1), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 3, 3), (3, 0, 1), (3, 2, 3), (3, 3, 1), (3, 3, 3)\}$
(28) 1, 5, 6	3
69) $R_{1,5,6;1}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 2, 0), (2, 3, 0), (2, 3, 1), (3, 0, 1),$

Table 1 (cont.)

Σ	Edges
	$(3, 2, 0), (3, 3, 0), (3, 3, 1), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 3, 1), (6, 0, 0), (6, 1, 0), (6, 1, 1), (6, 2, 1), (7, 0, 0), (7, 1, 0), (7, 1, 1), (7, 2, 1)$
70) $R_{1,5,6;2}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 0), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 3, 1), (6, 0, 0), (6, 2, 1), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$
71) $R_{1,5,6;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 1, 3), (0, 2, 0), (0, 2, 1), (0, 2, 2), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 3), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 3, 0), (1, 3, 2), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 2, 0), (2, 2, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 1), (3, 1, 3), (3, 2, 0), (3, 2, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2)\}$
(29) 1, 5, 7	3
72) $R_{1,5,7;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 2, 3), (0, 3, 1), (0, 3, 3), (1, 0, 3), (1, 1, 3), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 1), (3, 1, 1), (3, 2, 3), (3, 3, 3)\}$
73) $R_{1,5,7;2}$	$p_1 = 2, p_2 = 2, p_3 = 4$ $V_1 = \{(0, 0, 3), (0, 1, 3), (1, 0, 0), (1, 0, 1), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 3)\}$ $V_2 = \{(0, 1, 1), (0, 1, 2), (1, 1, 1), (1, 1, 2)\}$ $V_3 = \{(1, 0, 2), (1, 1, 2)\}$
74) $R_{1,5,7;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 2, 3), (0, 3, 1), (0, 3, 3), (1, 0, 3), (1, 1, 1), (1, 2, 1), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 1), (3, 1, 3), (3, 2, 3), (3, 3, 1)\}$
(30) 1, 5, 8	3
75) $R_{1,5,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 1), (1, 3, 0), (1, 3, 2), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 1), (3, 1, 0), (3, 1, 2), (3, 1, 3), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2)\}$
76) $R_{1,5,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 3, 2), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 2, 3), (3, 3, 0), (3, 3, 2), (3, 3, 3)\}$
77) $R_{1,5,8;3}$	$p_1 = 2, p_2 = 2, p_3 = 4$ $V_1 = \{(0, 0, 3), (0, 1, 3), (1, 0, 0), (1, 0, 1), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 3)\}$ $V_2 = \{(0, 0, 1), (0, 0, 3), (0, 1, 2), (0, 1, 3), (1, 0, 1), (1, 0, 3), (1, 1, 2), (1, 1, 3)\}$ $V_3 = \{(1, 0, 2), (1, 1, 2)\}$
(31) 1, 6, 6	2
78) $R_{1,6,6;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 3, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 0)\}$
79) $R_{1,6,6;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 3, 0), (3, 0, 1), (3, 2, 0), (3, 3, 0), (3, 3, 1)\}$
(32) 1, 6, 7	2
80) $R_{1,6,7;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 1), (3, 1, 1)\}$
81) $R_{1,6,7;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 1, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$
(33) 1, 6, 8	2
82) $R_{1,6,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 2, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0)\}$
83) $R_{1,6,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (2, 2, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0)\}$
(34) 1, 7, 7	3
84) $R_{1,7,7;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1)\}$
85) $R_{1,7,7;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 1, 1), (1, 2, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 3, 1)\}$
86) $R_{1,7,7;3}$	$p_1 = 4, p_2 = 2, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \{(0, 1, 2), (0, 1, 3), (1, 1, 2), (1, 1, 3), (2, 1, 2), (2, 1, 3), (3, 1, 2), (3, 1, 3)\}$ $V_3 = \{(2, 0, 0), (2, 0, 2), (2, 1, 0), (2, 1, 2), (3, 0, 0), (3, 0, 2), (3, 1, 0), (3, 1, 2)\}$
(35) 1, 7, 8	3
87) $R_{1,7,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$

Table 1 (cont.)

Σ	Edges
	$V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 1, 0), (3, 3, 0), (3, 3, 1)\}$
88) $R_{1,7,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (3, 0, 1), (3, 1, 0), (3, 3, 0), (3, 3, 1)\}$
89) $R_{1,7,8;3}$	$p_1 = 4, p_2 = 2, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \{(0, 0, 1), (0, 0, 2), (0, 1, 1), (0, 1, 3), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 1, 3), (2, 0, 1), (2, 0, 2), (2, 1, 1), (2, 1, 3), (3, 0, 1), (3, 0, 2), (3, 1, 1), (3, 1, 3)\}$ $V_3 = \{(2, 0, 0), (2, 0, 2), (2, 1, 0), (2, 1, 2), (3, 0, 0), (3, 0, 2), (3, 1, 0), (3, 1, 2)\}$
(36) 1, 8, 8	3
90) $R_{1,8,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$
91) $R_{1,8,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 2, 0), (3, 2, 1), (3, 3, 1)\}$
92) $R_{1,8,8;3}$	$p_1 = 4, p_2 = 2, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \{(0, 0, 2), (0, 0, 3), (0, 1, 1), (0, 1, 3), (1, 0, 2), (1, 0, 3), (1, 1, 1), (1, 1, 3), (2, 0, 2), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 1, 3)\}$ $V_3 = \{(1, 0, 0), (1, 0, 2), (1, 1, 0), (1, 1, 2), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 1, 3)\}$
(37) 2, 2, 2	1
93) $R_{2,2,2;1}$	$p_1 = 2, p_2 = 2, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0)\}$
(38) 2, 2, 3	1
94) $R_{2,2,3;1}$	$p_1 = 4, p_2 = 2, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 1, 0), (3, 0, 0), (3, 1, 0)\}$ $V_3 = \{(0, 0, 0), (1, 0, 0), (2, 1, 0), (3, 1, 0)\}$
(39) 2, 2, 4	1
95) $R_{2,2,4;1}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 4, 1), (1, 5, 0), (1, 5, 1), (1, 6, 0), (1, 6, 1), (1, 7, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1), (1, 5, 0), (1, 5, 1), (1, 6, 1), (1, 7, 0)\}$
(40) 2, 2, 5	1
96) $R_{2,2,5;1}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 4, 1), (1, 5, 0), (1, 5, 1), (1, 6, 0), (1, 6, 1), (1, 7, 0), (1, 7, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 7, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 6, 1), (1, 7, 1)\}$
(41) 2, 2, 6	1
97) $R_{2,2,6;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 3, 0)\}$
(42) 2, 2, 7	1
98) $R_{2,2,7;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0)\}$
(43) 2, 2, 8	1
99) $R_{2,2,8;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1)\}$
(44) 2, 3, 3	6
100) $R_{2,3,3;1}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 0), (3, 3, 0)\}$ $V_3 = \{(0, 2, 0), (0, 3, 0), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 1, 0), (3, 0, 0), (3, 1, 0)\}$
101) $R_{2,3,3;2}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 0), (3, 3, 0)\}$ $V_3 = \{(0, 1, 0), (0, 2, 0), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0)\}$
102) $R_{2,3,3;3}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 1, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0)\}$
103) $R_{2,3,3;4}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 1, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1)\}$
104) $R_{2,3,3;5}$	$p_1 = 2, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 0, 1)\}$
105) $R_{2,3,3;6}$	$p_1 = 2, p_2 = 2, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$ $V_3 = \{(0, 1, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$

Table 1 (cont.)

Σ	Edges
204) $R_{3,3,7;7}$	$p_1 = 1, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 0), (0, 0, 3), (0, 1, 0), (0, 1, 3), (0, 2, 1), (0, 2, 2), (0, 3, 1), (0, 3, 2)\}$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 2, 2), (0, 3, 0), (0, 3, 2)\}$
205) $R_{3,3,7;8}$	$p_1 = 1, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 0), (0, 0, 1), (0, 1, 2), (0, 1, 3), (0, 2, 2), (0, 2, 3), (0, 3, 0), (0, 3, 1)\}$ $V_2 = \emptyset$ $V_3 = \{(0, 2, 0), (0, 2, 2), (0, 3, 0), (0, 3, 2)\}$
206) $R_{3,3,7;9}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 2, 0), (0, 3, 0), (0, 3, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 2, 0)\}$
207) $R_{3,3,7;10}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 2, 0), (0, 3, 0), (0, 3, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0)\}$ $V_2 = \{(1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0)\}$ $V_3 = \{(0, 1, 1), (0, 2, 1), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1)\}$
(70) [3, 3, 8]	9
208) $R_{3,3,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1)\}$
209) $R_{3,3,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1)\}$
210) $R_{3,3,8;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(0, 0, 0), (0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 2, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 1), (3, 3, 1)\}$
211) $R_{3,3,8;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1)\}$ $V_3 = \{(0, 2, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 1), (3, 3, 1)\}$
212) $R_{3,3,8;5}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \{(0, 1, 0), (0, 3, 0), (1, 1, 0), (1, 3, 0), (2, 1, 0), (2, 3, 0), (3, 1, 0), (3, 3, 0)\}$ $V_2 = \{(2, 0, 0), (2, 2, 0), (3, 0, 0), (3, 2, 0)\}$ $V_3 = \{(0, 0, 0), (0, 1, 0), (1, 0, 0), (1, 1, 0), (2, 2, 0), (2, 3, 0), (3, 2, 0), (3, 3, 0)\}$
213) $R_{3,3,8;6}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \{(0, 1, 0), (0, 3, 0), (1, 1, 0), (1, 3, 0), (2, 1, 0), (2, 3, 0), (3, 1, 0), (3, 3, 0)\}$ $V_2 = \{(2, 0, 0), (2, 2, 0), (3, 0, 0), (3, 2, 0)\}$ $V_3 = \{(0, 0, 0), (0, 1, 0), (1, 2, 0), (1, 3, 0), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0)\}$
214) $R_{3,3,8;7}$	$p_1 = 4, p_2 = 4, p_3 = 1$ $V_1 = \{(0, 1, 0), (0, 3, 0), (1, 1, 0), (1, 3, 0), (2, 1, 0), (2, 3, 0), (3, 1, 0), (3, 3, 0)\}$ $V_2 = \{(2, 0, 0), (2, 2, 0), (3, 0, 0), (3, 2, 0)\}$ $V_3 = \{(0, 0, 0), (0, 3, 0), (1, 0, 0), (1, 3, 0), (2, 1, 0), (2, 2, 0), (3, 1, 0), (3, 2, 0)\}$
215) $R_{3,3,8;8}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 2, 0), (0, 3, 0), (0, 3, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 1), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1)\}$
216) $R_{3,3,8;9}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 2, 0), (0, 3, 0), (0, 3, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0)\}$ $V_2 = \{(1, 0, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 2, 0)\}$
(71) [3, 4, 4]	12
217) $R_{3,4,4;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \{(1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2), (1, 3, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2)\}$
218) $R_{3,4,4;2}$	$p_1 = 8, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 0, 2), (2, 0, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 0), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3), (6, 0, 0), (6, 0, 1), (6, 0, 2), (6, 0, 3), (6, 1, 0), (6, 1, 1), (6, 1, 2), (6, 1, 3), (6, 2, 0), (6, 2, 1), (6, 2, 2), (6, 2, 3), (6, 3, 0), (6, 3, 1), (6, 3, 2), (6, 3, 3), (7, 0, 0), (7, 0, 1), (7, 0, 2), (7, 0, 3), (7, 1, 0), (7, 1, 1), (7, 1, 2), (7, 1, 3), (7, 2, 0), (7, 2, 1), (7, 2, 2), (7, 2, 3), (7, 3, 0), (7, 3, 1), (7, 3, 2), (7, 3, 3)\}$ $V_3 = \{(0, 1, 0), (0, 2, 0), (0, 2, 2), (0, 3, 2), (1, 0, 1), (1, 0, 3), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 0), (2, 0, 0), (2, 0, 2), (2, 1, 0), (2, 1, 1), (2, 1, 3), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 1, 0), (3, 2, 0), (3, 2, 2), (3, 3, 2), (4, 0, 0), (4, 0, 1), (4, 0, 2), (4, 0, 3), (4, 1, 1), (4, 1, 2), (4, 1, 3), (4, 2, 1), (4, 2, 3), (4, 3, 0), (4, 3, 1), (4, 3, 3), (5, 0, 0), (5, 0, 2), (5, 1, 0), (5, 1, 1), (5, 1, 3), (5, 3, 1), (5, 3, 2), (5, 3, 3), (6, 0, 1), (6, 0, 3), (6, 1, 2), (6, 2, 0), (6, 2, 1), (6, 2, 2), (6, 2, 3), (6, 3, 0), (7, 0, 0), (7, 0, 1), (7, 0, 2), (7, 0, 3), (7, 1, 1), (7, 1, 2), (7, 1, 3), (7, 2, 1), (7, 2, 2), (7, 2, 3), (7, 3, 0), (7, 3, 1), (7, 3, 3)\}$
219) $R_{3,4,4;3}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 2), (0, 0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 2), (0, 2, 3), (0, 3, 2), (0, 3, 3), (1, 0, 2), (1, 0, 3), (1, 1, 2), (1, 1, 3), (1, 2, 2), (1, 2, 3), (1, 3, 2), (1, 3, 3), (2, 0, 2), (2, 0, 3), (2, 1, 2), (2, 1, 3), (2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 0, 2), (3, 0, 3), (3, 1, 2), (3, 1, 3), (3, 2, 2), (3, 2, 3), (3, 3, 2), (3, 3, 3)\}$ $V_2 = \{(0, 0, 2), (0, 0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 2), (0, 2, 3), (0, 3, 2), (0, 3, 3), (1, 0, 2), (1, 0, 3), (1, 1, 2), (1, 1, 3), (1, 2, 2), (1, 2, 3), (1, 3, 2), (1, 3, 3), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 2, 1), (0, 2, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 0), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2)\}$
220) $R_{3,4,4;4}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \emptyset$

Table 1 (cont.)

Σ	Edges
	$V_2 = \{(1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$ $V_3 = \{(0, 2, 0), (0, 2, 1), (0, 2, 2), (0, 2, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 2), (1, 0, 3), (1, 1, 0), (1, 1, 2), (1, 1, 3), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (2, 2, 0), (2, 2, 2), (2, 3, 0), (2, 3, 2), (3, 0, 0), (3, 0, 1), (3, 0, 2), (3, 1, 0), (3, 1, 1), (3, 1, 2), (3, 2, 3), (3, 3, 3)\}$
221 $R_{3,4,4;5}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (5, 2, 1), (6, 1, 0), (6, 1, 1), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 0, 1), (7, 2, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_3 = \{(0, 2, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 1), (3, 3, 1), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (5, 2, 0), (5, 2, 1), (5, 3, 0), (5, 3, 1), (6, 0, 0), (6, 1, 0), (6, 2, 1), (6, 3, 1), (7, 0, 1), (7, 1, 1), (7, 2, 0), (7, 3, 0)\}$
222 $R_{3,4,4;6}$	$p_1 = 8, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 2), (0, 0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 2), (0, 2, 3), (0, 3, 2), (0, 3, 3), (1, 0, 2), (1, 0, 3), (1, 1, 2), (1, 1, 3), (1, 2, 2), (1, 2, 3), (1, 3, 2), (1, 3, 3), (2, 0, 2), (2, 0, 3), (2, 1, 2), (2, 1, 3), (2, 2, 2), (2, 2, 3), (2, 3, 2), (2, 3, 3), (3, 0, 2), (3, 0, 3), (3, 1, 2), (3, 1, 3), (3, 2, 2), (3, 2, 3), (3, 3, 2), (3, 3, 3), (4, 0, 2), (4, 0, 3), (4, 1, 2), (4, 1, 3), (4, 2, 2), (4, 2, 3), (4, 3, 2), (4, 3, 3), (5, 0, 2), (5, 0, 3), (5, 1, 2), (5, 1, 3), (5, 2, 2), (5, 2, 3), (5, 3, 2), (5, 3, 3), (6, 0, 2), (6, 0, 3), (6, 1, 2), (6, 1, 3), (6, 2, 2), (6, 2, 3), (6, 3, 2), (6, 3, 3), (7, 0, 2), (7, 0, 3), (7, 1, 2), (7, 1, 3), (7, 2, 2), (7, 2, 3), (7, 3, 2), (7, 3, 3)\}$ $V_2 = \{(0, 0, 2), (0, 0, 3), (0, 1, 2), (0, 1, 3), (0, 2, 2), (0, 2, 3), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 2), (1, 3, 3), (2, 0, 2), (2, 0, 3), (2, 1, 0), (2, 1, 1), (2, 2, 2), (2, 2, 3), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 1, 2), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1), (4, 0, 2), (4, 0, 3), (4, 1, 2), (4, 1, 3), (4, 2, 2), (4, 2, 3), (4, 3, 2), (4, 3, 3), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (5, 2, 1), (5, 3, 2), (5, 3, 3), (6, 0, 2), (6, 0, 3), (6, 1, 0), (6, 1, 1), (6, 2, 2), (6, 2, 3), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 0, 1), (7, 1, 2), (7, 1, 3), (7, 2, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 0, 1), (2, 0, 3), (2, 1, 1), (2, 1, 3), (2, 2, 1), (2, 2, 3), (2, 3, 1), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 1, 0), (3, 1, 2), (3, 2, 0), (3, 2, 2), (3, 3, 0), (3, 3, 2), (4, 0, 0), (4, 0, 1), (4, 0, 2), (4, 0, 3), (4, 1, 0), (4, 1, 1), (4, 1, 2), (4, 1, 3), (4, 2, 0), (4, 2, 1), (4, 2, 2), (4, 2, 3), (4, 3, 0), (4, 3, 1), (4, 3, 2), (4, 3, 3), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (5, 2, 1), (5, 3, 2), (5, 3, 3), (6, 0, 0), (6, 0, 2), (6, 1, 0), (6, 1, 1), (6, 2, 2), (6, 2, 3), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 0, 1), (7, 1, 2), (7, 1, 3), (7, 2, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$
223 $R_{3,4,4;7}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 2, 1), (1, 3, 0)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
224 $R_{3,4,4;8}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 2, 1), (1, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
225 $R_{3,4,4;9}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 3, 1), (2, 1, 1), (2, 2, 1)\}$ $V_2 = \{(0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
226 $R_{3,4,4;10}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
227 $R_{3,4,4;11}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 1), (2, 1, 1), (2, 2, 1), (3, 0, 1), (3, 1, 1), (3, 2, 1), (3, 3, 1)\}$ $V_2 = \{(0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
228 $R_{3,4,4;12}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 3, 0), (1, 0, 1), (1, 1, 0), (1, 2, 1), (1, 3, 0), (2, 1, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 1, 0), (3, 2, 1), (3, 3, 0)\}$ $V_2 = \{(0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
(72) [3, 4, 5]	4
229 $R_{3,4,5;1}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 3, 0)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
230 $R_{3,4,5;2}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (1, 0, 0), (1, 1, 1), (1, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
231 $R_{3,4,5;3}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 2, 0), (0, 2, 1), (0, 3, 1), (1, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
232 $R_{3,4,5;4}$	$p_1 = 2, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 2, 0), (1, 2, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (1, 0, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1)\}$
(73) [3, 4, 6]	6
233 $R_{3,4,6;1}$	$p_1 = 2, p_2 = 4, p_3 = 8$ $V_1 = \{(0, 0, 1), (0, 0, 3), (0, 0, 4), (0, 0, 6), (0, 1, 1), (0, 1, 3), (0, 1, 4), (0, 1, 6), (0, 2, 0), (0, 2, 2), (0, 2, 5), (0, 2, 7), (0, 3, 0), (0, 3, 2), (0, 3, 5), (0, 3, 7), (1, 0, 1), (1, 0, 2), (1, 0, 4), (1, 0, 7), (1, 1, 1), (1, 1, 2), (1, 1, 4), (1, 1, 7), (1, 2, 0), (1, 2, 3), (1, 2, 5), (1, 2, 6), (1, 3, 0), (1, 3, 3), (1, 3, 5), (1, 3, 6)\}$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 1, 3), (0, 1, 5), (0, 1, 7), (0, 2, 0), (0, 2, 1), (0, 2, 2), (0, 2, 3), (0, 2, 4), (0, 2, 5), (0, 2, 6), (0, 2, 7), (0, 3, 0), (0, 3, 2), (0, 3, 4), (0, 3, 6), (1, 1, 1), (1, 1, 3), (1, 1, 5), (1, 1, 7), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 2, 7), (1, 3, 0), (1, 3, 2), (1, 3, 4), (1, 3, 6)\}$
234 $R_{3,4,6;2}$	$p_1 = 2, p_2 = 4, p_3 = 8$ $V_1 = \{(0, 0, 1), (0, 0, 3), (0, 0, 4), (0, 0, 6), (0, 1, 0), (0, 1, 2), (0, 1, 5), (0, 1, 7), (0, 2, 0), (0, 2, 2), (0, 2, 5), (0, 2, 7), (0, 3, 1), (0, 3, 3), (0, 3, 4), (0, 3, 6), (1, 0, 1), (1, 0, 2), (1, 0, 4), (1, 0, 7), (1, 1, 0), (1, 1, 3), (1, 1, 5), (1, 1, 6), (1, 2, 0), (1, 2, 3), (1, 2, 5), (1, 2, 6), (1, 3, 1), (1, 3, 2), (1, 3, 4), (1, 3, 7)\}$ $V_2 = \emptyset$ $V_3 = \{(0, 1, 1), (0, 1, 3), (0, 1, 5), (0, 1, 7), (0, 2, 0), (0, 2, 1), (0, 2, 2), (0, 2, 3), (0, 2, 4), (0, 2, 5), (0, 2, 6), (0, 2, 7), (0, 3, 0), (0, 3, 2), (0, 3, 4), (0, 3, 6), (1, 1, 1), (1, 1, 3), (1, 1, 5), (1, 1, 7), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 2, 4), (1, 2, 5), (1, 2, 6), (1, 2, 7), (1, 3, 0), (1, 3, 2), (1, 3, 4), (1, 3, 6)\}$
235 $R_{3,4,6;3}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$

Table 1 (cont.)

Σ	Edges
$R_{7,7,8;4}$	$V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 1, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 1, 0), (3, 3, 0), (3, 3, 1)\}$
351) $R_{7,7,8;5}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 1), (1, 1, 0), (1, 3, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 2, 1), (3, 3, 0)\}$
352) $R_{7,7,8;6}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1), (2, 0, 1), (2, 1, 1), (3, 2, 1), (3, 3, 1)\}$
(101) 7, 8, 8	6
353) $R_{7,8,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 1, 1), (3, 2, 0), (3, 2, 1)\}$
354) $R_{7,8,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (3, 0, 0), (3, 2, 0), (3, 2, 1), (3, 3, 1)\}$
355) $R_{7,8,8;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (2, 0, 1), (2, 1, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1)\}$
356) $R_{7,8,8;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 3, 1), (1, 0, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 1), (3, 3, 0)\}$
357) $R_{7,8,8;5}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$
358) $R_{7,8,8;6}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \emptyset$ $V_2 = \{(0, 1, 1), (0, 3, 1), (1, 1, 1), (1, 3, 1), (2, 1, 0), (2, 3, 0), (3, 1, 0), (3, 3, 0)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (2, 1, 0), (2, 1, 1), (2, 2, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 3, 1)\}$
(102) 8, 8, 8	4
359) $R_{8,8,8;1}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (3, 0, 0), (3, 2, 0), (3, 2, 1), (3, 3, 1)\}$
360) $R_{8,8,8;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0)\}$
361) $R_{8,8,8;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 2, 0), (0, 3, 1), (1, 0, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 1), (3, 3, 0)\}$
362) $R_{8,8,8;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 3, 1), (2, 1, 0), (2, 1, 1), (2, 3, 0), (2, 3, 1), (3, 1, 0), (3, 1, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 2, 0), (2, 2, 1), (3, 0, 0), (3, 0, 1), (3, 2, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (1, 3, 1), (2, 0, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 2, 1)\}$

5. The List of Layer Non-Decomposable Saturated Realizations of Symmetrical 2-Extensions of the Grid Λ^3 with All Connections of Type 2

Recall (see [Introduction](#)) that to determine a realization $(\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^3 up to equivalence it is sufficient to determine Γ, σ and φ .

Table 2 below is structured using descriptors $[n_1, n_2, n_3]$, which are lexicographically numerated by (103), (104), ..., (107) (the numbering continues from Table 1). Namely, each “heading” line of Table 2 contains a descriptor number (s) (where $s \in \{103, \dots, 107\}$), the descriptor $[n_1, n_2, n_3]$ in the first column, and the number of realizations corresponding to this descriptor in the second column. Since Table 2 contains layer non-decomposable realizations, some of n_1, n_2, n_3 are specified as “-” (“-” succeeds any positive integer in our lexicographical order). Between a heading line and the next

one all realizations coresponding to $[n_1, n_2, n_3]$ are described. In the first column, the number n ($n \in \{363, 364, \dots, 480\}$, the numbering continues from Table 1) of realization and the realization name $R_{n_1, n_2, n_3; m}$ are specified. The index m is used to number non-equivalent realizations, having the same descriptor. In the second column the edge set of the graph of the realization is given by periods p_1, p_2, p_3 and sets V_1, V_2, V_3 which are defined as follows.

For the graph Γ of a saturated realization $R = (\Gamma, G, \sigma, \varphi)$ of a symmetrical 2-extension of the grid Λ^3 with all connections of type 2, we put

$$V(\Gamma) = \{(i, j, k, l) : i, j, k \in \mathbb{Z}, l \in \{1, 2\}\}$$

with

$$\sigma = \{(i, j, k, 1), (i, j, k, 2)\} : i, j, k \in \mathbb{Z}\}$$

and

$$\varphi : \{(i, j, k, 1), (i, j, k, 2)\} \mapsto (i, j, k)$$

for $i, j, k \in \mathbb{Z}$.

To specify the edge set of Γ , we use the following notation:

$$E(\Gamma) = E_0(\Gamma) \cup E_1(\Gamma) \cup E_2(\Gamma) \cup E_3(\Gamma),$$

where

$$\begin{aligned} E_0(\Gamma) &= \{(i, j, k, l_1), (i, j, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}\}, \\ E_1(\Gamma) &\subseteq \{(i, j, k, l_1), (i+1, j, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}\}, \\ E_2(\Gamma) &\subseteq \{(i, j, k, l_1), (i, j+1, k, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}\}, \\ E_3(\Gamma) &\subseteq \{(i, j, k, l_1), (i, j, k+1, l_2)\} : i, j, k \in \mathbb{Z}, l_1, l_2 \in \{1, 2\}\}. \end{aligned}$$

To describe $E_1(\Gamma)$, $E_2(\Gamma)$, $E_3(\Gamma)$, it is enough to define the sets

$$\begin{aligned} W_1(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i+1, j, k, 2)\} \in E_1(\Gamma)\}, \\ W_2(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i, j+1, k, 2)\} \in E_2(\Gamma)\}, \\ W_3(\Gamma) &:= \{(i, j, k) : i, j, k \in \mathbb{Z}, \{(i, j, k, 1), (i, j, k+1, 2)\} \in E_3(\Gamma)\}. \end{aligned}$$

In fact

$$\begin{aligned} E_1(\Gamma) &= \{(i, j, k, 1), (i+1, j, k, 2)\}, \{(i, j, k, 2), (i+1, j, k, 1)\} : (i, j, k) \in W_1(\Gamma)\} \\ &\cup \{(i, j, k, 1), (i+1, j, k, 1)\}, \{(i, j, k, 2), (i+1, j, k, 2)\} : (i, j, k) \notin W_1(\Gamma)\}, \\ E_2(\Gamma) &= \{(i, j, k, 1), (i, j+1, k, 2)\}, \{(i, j, k, 2), (i, j+1, k, 1)\} : (i, j, k) \in W_2(\Gamma)\} \\ &\cup \{(i, j, k, 1), (i, j+1, k, 1)\}, \{(i, j, k, 2), (i, j+1, k, 2)\} : (i, j, k) \notin W_2(\Gamma)\}, \\ E_3(\Gamma) &= \{(i, j, k, 1), (i, j, k+1, 2)\}, \{(i, j, k, 2), (i, j, k+1, 1)\} : (i, j, k) \in W_3(\Gamma)\} \\ &\cup \{(i, j, k, 1), (i, j, k+1, 1)\}, \{(i, j, k, 2), (i, j, k+1, 2)\} : (i, j, k) \notin W_3(\Gamma)\}. \end{aligned}$$

The sets $W_1(\Gamma)$, $W_2(\Gamma)$ and $W_3(\Gamma)$ are periodic with some positive periods p_1, p_2 and p_3 (for the concept of $[p_1, p_2, p_3]$ -periodicity of a realization, see [2]). It means, that

$$\begin{aligned} W_1(\Gamma) &= \{(i+ap_1, j+bp_2, k+cp_3) : (i, j, k) \in V_1(\Gamma), a, b, c \in \mathbb{Z}\}, \\ W_2(\Gamma) &= \{(i+ap_1, j+bp_2, k+cp_3) : (i, j, k) \in V_2(\Gamma), a, b, c \in \mathbb{Z}\}, \\ W_3(\Gamma) &= \{(i+ap_1, j+bp_2, k+cp_3) : (i, j, k) \in V_3(\Gamma), a, b, c \in \mathbb{Z}\}, \end{aligned}$$

for some subsets $V_1 = V_1(\Gamma)$, $V_2 = V_2(\Gamma)$, $V_3 = V_3(\Gamma)$ of the set

$$\{(i, j, k) : i \in \{0, 1, \dots, p_1 - 1\}, j \in \{0, 1, \dots, p_2 - 1\}, k \in \{0, 1, \dots, p_3 - 1\}\}.$$

Table 2. 118 layer non-decomposable saturated realizations of symmetrical 2-extensions of the grid Λ^3 with all connections of type 2

(103) 1, -, -	20
363) $R_{1,-,-;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \{(1, 1, 1), (1, 1, 3), (1, 3, 1), (1, 3, 3), (3, 1, 1), (3, 1, 3), (3, 3, 1), (3, 3, 3)\}$ $V_2 = \{(0, 0, 3), (0, 1, 3), (0, 2, 1), (0, 3, 1), (1, 0, 3), (1, 1, 3), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 1, 1), (2, 2, 3), (2, 3, 3), (3, 0, 1), (3, 1, 1), (3, 2, 3), (3, 3, 3)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (0, 1, 2), (0, 1, 3), (0, 2, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 3), (1, 2, 1), (1, 2, 2), (1, 2, 3), (1, 3, 1), (1, 3, 2), (1, 3, 3), (2, 0, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 2, 1), (2, 3, 0), (2, 3, 2), (2, 3, 3), (3, 0, 1), (3, 0, 2), (3, 0, 3), (3, 1, 1), (3, 1, 2), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 3)\}$
364) $R_{1,-,-;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1), (2, 0, 1), (2, 1, 1), (2, 2, 1), (2, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 1), (1, 0, 1), (1, 2, 1), (2, 0, 1), (2, 2, 1), (3, 0, 1), (3, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
365) $R_{1,-,-;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 1, 1), (2, 2, 1), (2, 3, 1), (3, 0, 1), (3, 1, 1), (3, 2, 1), (3, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 1), (1, 0, 1), (1, 2, 1), (2, 0, 1), (2, 2, 1), (3, 0, 1), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
366) $R_{1,-,-;4}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 1), (3, 0, 1), (3, 1, 0), (5, 1, 0), (5, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 0, 1), (3, 1, 0), (3, 1, 1), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (5, 0, 1), (5, 1, 1), (6, 0, 1), (6, 1, 1)\}$
367) $R_{1,-,-;5}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$
368) $R_{1,-,-;6}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 1), (5, 1, 0), (5, 1, 1), (7, 0, 1), (7, 1, 0)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (3, 1, 0), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 1, 1)\}$ $V_3 = \{(1, 0, 1), (1, 1, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1)\}$
369) $R_{1,-,-;7}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (1, 1, 1), (5, 0, 1), (5, 1, 1), (7, 0, 1), (7, 1, 0)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 0, 1), (3, 1, 0), (3, 1, 1), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (5, 0, 1), (5, 1, 1), (6, 0, 1), (6, 1, 1)\}$
370) $R_{1,-,-;8}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 0)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1)\}$
371) $R_{1,-,-;9}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (3, 1, 0), (3, 1, 1), (7, 0, 1), (7, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 1, 1), (7, 1, 0), (7, 1, 1)\}$ $V_3 = \{(1, 0, 1), (1, 1, 1), (4, 0, 0), (4, 1, 0), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (6, 0, 1), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$
372) $R_{1,-,-;10}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(3, 1, 0), (3, 1, 1), (5, 0, 1), (5, 1, 0), (7, 0, 1), (7, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (4, 0, 0), (4, 1, 0), (5, 0, 1), (5, 1, 1), (7, 0, 1), (7, 1, 1)\}$
373) $R_{1,-,-;11}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (3, 1, 0), (3, 1, 1)\}$ $V_3 = \{(1, 0, 0), (1, 1, 0), (3, 0, 1), (3, 1, 1)\}$
374) $R_{1,-,-;12}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 2, 0), (0, 3, 0), (0, 4, 1), (0, 5, 1), (1, 2, 1), (1, 3, 1), (1, 4, 0), (1, 5, 0)\}$ $V_2 = \{(0, 3, 0), (0, 3, 1), (0, 5, 0), (1, 1, 0)\}$ $V_3 = \{(0, 4, 1), (0, 5, 1), (0, 6, 0), (0, 7, 0), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0), (1, 6, 0), (1, 7, 0)\}$
375) $R_{1,-,-;13}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 2, 0), (0, 3, 0), (0, 4, 1), (0, 5, 0), (0, 6, 0), (0, 6, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0)\}$ $V_2 = \{(0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 1, 0), (1, 1, 1), (1, 5, 0)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 4, 1), (0, 7, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0)\}$
376) $R_{1,-,-;14}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 1), (0, 5, 1), (1, 2, 1), (1, 3, 1), (1, 4, 0), (1, 5, 1), (1, 6, 0), (1, 6, 1)\}$ $V_2 = \{(0, 1, 0), (0, 3, 0), (0, 3, 1), (0, 5, 0), (0, 5, 1), (1, 5, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 6, 1), (0, 7, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 6, 0), (1, 6, 1), (1, 7, 0)\}$
377) $R_{1,-,-;15}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 4, 0), (0, 5, 0), (0, 6, 1), (0, 7, 1), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 4, 0), (1, 5, 1), (1, 6, 0), (1, 7, 1)\}$ $V_2 = \{(0, 3, 0), (0, 7, 1), (1, 3, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 3, 1), (0, 4, 0), (0, 4, 1), (0, 5, 1), (0, 6, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 2, 0), (1, 5, 0), (1, 6, 0), (1, 7, 0), (1, 7, 1)\}$
378) $R_{1,-,-;16}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 2, 0), (0, 3, 0), (0, 4, 1), (0, 5, 0), (0, 6, 0), (0, 6, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0)\}$ $V_2 = \{(0, 3, 0), (0, 5, 1), (0, 7, 1), (1, 1, 1), (1, 3, 0), (1, 7, 0)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 4, 1), (0, 7, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0)\}$
379) $R_{1,-,-;17}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 4, 1), (0, 5, 1), (1, 2, 1), (1, 3, 1), (1, 4, 0), (1, 5, 1), (1, 6, 0), (1, 6, 1)\}$ $V_2 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 5, 0), (0, 7, 1), (1, 1, 0), (1, 3, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 6, 1), (0, 7, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 6, 0), (1, 6, 1), (1, 7, 0)\}$
380) $R_{1,-,-;18}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 0)\}$ $V_2 = \{(2, 1, 0), (2, 1, 1), (3, 0, 0), (3, 0, 1)\}$ $V_3 = \{(0, 0, 1), (0, 1, 1), (1, 0, 0), (1, 1, 0)\}$
381) $R_{1,-,-;19}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (5, 0, 0), (5, 1, 0), (7, 0, 1), (7, 1, 0)\}$ $V_2 = \{(2, 0, 0), (2, 0, 1), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 1, 1), (7, 0, 0), (7, 0, 1)\}$ $V_3 = \{(0, 0, 1), (0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (5, 0, 1), (5, 1, 1)\}$
382) $R_{1,-,-;20}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (1, 0, 1), (1, 1, 1), (2, 0, 1), (2, 1, 0), (4, 0, 1), (4, 1, 0), (5, 1, 0), (5, 1, 1), (6, 0, 1), (6, 1, 0), (7, 0, 1), (7, 1, 0)\}$ $V_2 = \{(2, 1, 0), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (7, 0, 0), (7, 0, 1)\}$

Table 2 (cont.)

Σ	Edges
	$V_3 = \{(0, 0, 1), (0, 1, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (5, 0, 0), (5, 1, 0)\}$
(104) 2, -, -	20
383 $R_{2,-,-;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 1), (0, 1, 3), (0, 2, 3), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 2, 2), (1, 3, 2), (2, 0, 3), (2, 1, 1), (2, 2, 1), (2, 3, 3), (3, 0, 2), (3, 1, 2), (3, 2, 0), (3, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 2, 3), (0, 3, 2), (1, 0, 0), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 1, 2), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 2, 3), (1, 3, 0), (1, 3, 1), (1, 3, 3), (2, 0, 3), (2, 1, 2), (2, 2, 1), (2, 3, 0), (3, 0, 0), (3, 0, 2), (3, 0, 3), (3, 1, 0), (3, 1, 1), (3, 1, 3), (3, 2, 0), (3, 2, 1), (3, 2, 2), (3, 3, 1), (3, 3, 2), (3, 3, 3)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (0, 3, 3), (1, 0, 0), (1, 0, 2), (1, 1, 1), (1, 1, 3), (1, 2, 0), (1, 2, 2), (1, 3, 1), (1, 3, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 2), (3, 1, 1), (3, 1, 3), (3, 2, 0), (3, 2, 2), (3, 3, 1), (3, 3, 3)\}$
384 $R_{2,-,-;2}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 0), (2, 2, 1), (2, 3, 0), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 0), (1, 0, 1), (1, 2, 0), (2, 0, 0), (2, 2, 1), (3, 0, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
385 $R_{2,-,-;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (0, 3, 0), (1, 0, 0), (1, 1, 0), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 2, 1), (2, 3, 0), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 2, 0), (1, 0, 1), (1, 2, 0), (2, 0, 0), (2, 2, 1), (3, 0, 0), (3, 2, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
386 $R_{2,-,-;4}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (5, 0, 1), (7, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 0), (3, 0, 1), (3, 1, 0), (4, 0, 0), (4, 0, 1), (5, 0, 1), (6, 0, 0), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0), (2, 0, 1), (2, 0, 2), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 1), (5, 0, 0), (5, 0, 1)\}$
387 $R_{2,-,-;5}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (3, 0, 1), (3, 1, 0), (3, 1, 1), (5, 1, 0), (7, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 1, 1), (3, 0, 1), (3, 1, 0), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 1, 0), (7, 0, 1), (7, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 1), (5, 0, 1), (5, 1, 0), (6, 0, 1), (6, 1, 1)\}$
388 $R_{2,-,-;6}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (3, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 0), (3, 0, 1), (3, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1)\}$
389 $R_{2,-,-;7}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (3, 1, 1), (5, 1, 0), (7, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 0), (3, 0, 1), (3, 1, 0), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 0), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 1), (5, 0, 0), (5, 0, 1)\}$
390 $R_{2,-,-;8}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (3, 1, 1), (5, 0, 1), (7, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 1, 1), (3, 0, 1), (3, 1, 0), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 1, 0), (7, 0, 1), (7, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 1), (5, 0, 1), (5, 1, 0), (6, 0, 1), (6, 1, 1)\}$
391 $R_{2,-,-;9}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 0), (3, 0, 1), (3, 1, 1)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1)\}$
392 $R_{2,-,-;10}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 1, 0), (5, 1, 1), (7, 0, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 0), (6, 1, 0), (7, 0, 1), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 1), (1, 1, 0), (4, 0, 0), (4, 1, 1), (5, 0, 0), (5, 0, 1), (6, 0, 1), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$
393 $R_{2,-,-;11}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 1), (3, 1, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (7, 0, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 0), (2, 1, 0), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (6, 0, 0), (6, 1, 1), (7, 0, 1), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 0, 1), (2, 0, 1), (2, 1, 1), (4, 0, 0), (4, 1, 1), (5, 0, 1), (5, 1, 0), (7, 0, 1), (7, 1, 1)\}$
394 $R_{2,-,-;12}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (3, 0, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 0)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (1, 0, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1)\}$
395 $R_{2,-,-;13}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 0), (0, 3, 0), (0, 4, 0), (0, 5, 1), (0, 6, 0), (0, 7, 1), (1, 0, 1), (1, 1, 0), (1, 2, 1), (1, 3, 0), (1, 4, 0), (1, 5, 0), (1, 6, 1), (1, 7, 1)\}$ $V_2 = \{(0, 1, 1), (0, 3, 0), (0, 5, 0), (0, 7, 1), (1, 1, 1), (1, 3, 0), (1, 5, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 5, 1), (0, 6, 0), (1, 1, 0), (1, 2, 1), (1, 5, 1), (1, 6, 0)\}$
396 $R_{2,-,-;14}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 1), (1, 1, 0), (1, 3, 0), (1, 3, 1), (1, 6, 1), (1, 7, 1)\}$ $V_2 = \{(0, 1, 1), (0, 3, 1), (0, 7, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (1, 5, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 4, 1), (0, 7, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0)\}$
397 $R_{2,-,-;15}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 1), (0, 7, 1), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 5, 0), (1, 5, 1), (1, 6, 0), (1, 7, 1)\}$ $V_2 = \{(0, 1, 0), (0, 3, 1), (0, 5, 0), (0, 5, 1), (0, 7, 1), (1, 3, 0), (1, 5, 1), (1, 7, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 6, 1), (0, 7, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 6, 0), (1, 6, 1), (1, 7, 0)\}$
398 $R_{2,-,-;16}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 4, 0), (0, 4, 1), (0, 6, 0), (0, 7, 1), (1, 0, 1), (1, 1, 0), (1, 3, 0), (1, 3, 1), (1, 6, 1), (1, 7, 1)\}$ $V_2 = \{(0, 5, 1), (1, 1, 1), (1, 7, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 4, 1), (0, 7, 0), (1, 1, 0), (1, 2, 0), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0)\}$
399 $R_{2,-,-;17}$	$p_1 = 2, p_2 = 8, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 0), (0, 2, 1), (0, 4, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 1), (0, 7, 1), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 5, 0), (1, 5, 1), (1, 6, 0), (1, 7, 1)\}$ $V_2 = \{(0, 1, 0), (0, 1, 1), (0, 5, 0), (1, 1, 0), (1, 5, 0), (1, 5, 1), (1, 7, 0), (1, 7, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (0, 4, 1), (0, 5, 0), (0, 5, 1), (0, 6, 0), (0, 6, 1), (0, 7, 0), (1, 1, 1), (1, 2, 1), (1, 3, 0), (1, 3, 1), (1, 4, 0), (1, 6, 0), (1, 6, 1), (1, 7, 0)\}$
400 $R_{2,-,-;18}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1), (5, 0, 1), (7, 1, 1)\}$ $V_2 = \{(2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 1, 0), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 0, 1), (6, 0, 1), (6, 1, 1), (7, 0, 0), (7, 1, 1)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (1, 1, 0), (1, 1, 1), (2, 0, 1), (2, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 0, 0), (5, 1, 1)\}$
401 $R_{2,-,-;19}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (3, 0, 1), (3, 1, 0), (3, 1, 1), (5, 1, 0), (7, 1, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (2, 0, 1), (2, 1, 1), (3, 0, 0), (3, 1, 0), (4, 0, 0), (4, 0, 1), (6, 0, 1), (6, 1, 0), (7, 0, 0), (7, 1, 1)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (5, 1, 0), (5, 1, 1), (6, 0, 1), (6, 1, 1)\}$
402 $R_{2,-,-;20}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (3, 1, 1)\}$ $V_2 = \{(2, 0, 1), (2, 1, 0), (3, 0, 0), (3, 1, 1)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (1, 0, 0), (1, 1, 1)\}$

Table 2 (cont.)

Σ	Edges
(105) 6, -, -	26
403) $R_{6,-,-;1}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 1), (0, 0, 2), (0, 1, 3), (0, 2, 0), (0, 2, 3), (0, 3, 1), (1, 0, 0), (1, 0, 3), (1, 1, 1), (1, 2, 1), (1, 2, 2), (1, 3, 3), (2, 0, 0), (2, 0, 3), (2, 1, 1), (2, 2, 1), (2, 2, 2), (2, 3, 3), (3, 0, 1), (3, 0, 2), (3, 1, 3), (3, 2, 0), (3, 2, 3), (3, 3, 1)\}$ $V_2 = \{(0, 0, 3), (0, 1, 3), (0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 0, 2), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 3), (1, 3, 0), (1, 3, 3), (2, 0, 1), (2, 1, 1), (2, 2, 3), (2, 3, 3), (3, 0, 0), (3, 0, 3), (3, 1, 0), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 3, 1), (3, 3, 2)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (0, 1, 2), (0, 1, 3), (0, 2, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (1, 0, 1), (1, 0, 2), (1, 0, 3), (1, 1, 0), (1, 1, 1), (1, 1, 2), (1, 2, 0), (1, 2, 1), (1, 2, 2), (1, 2, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 2, 0), (2, 2, 1), (2, 2, 2), (2, 2, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 1), (3, 0, 3), (3, 1, 0), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2)\}$
404) $R_{6,-,-;2}$	$p_1 = 4, p_2 = 4, p_3 = 4$ $V_1 = \{(0, 0, 2), (0, 2, 0), (1, 0, 3), (1, 1, 0), (1, 1, 3), (1, 2, 1), (1, 3, 1), (1, 3, 2), (2, 0, 0), (2, 2, 2), (3, 0, 1), (3, 1, 1), (3, 1, 2), (3, 2, 3), (3, 3, 0), (3, 3, 3)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 2, 3), (0, 3, 2), (1, 0, 0), (1, 0, 3), (1, 2, 1), (1, 2, 2), (2, 0, 3), (2, 1, 2), (2, 2, 1), (2, 3, 0), (3, 0, 1), (3, 0, 2), (3, 2, 0), (3, 2, 3)\}$ $V_3 = \{(0, 1, 0), (0, 1, 1), (0, 1, 2), (0, 1, 3), (0, 3, 0), (0, 3, 1), (0, 3, 2), (0, 3, 3), (1, 1, 1), (1, 1, 2), (1, 3, 0), (1, 3, 3), (2, 1, 0), (2, 1, 1), (2, 1, 2), (2, 1, 3), (2, 3, 0), (2, 3, 1), (2, 3, 2), (2, 3, 3), (3, 0, 0), (3, 0, 1), (3, 0, 3), (3, 1, 0), (3, 1, 2), (3, 1, 3), (3, 2, 1), (3, 2, 2), (3, 2, 3), (3, 3, 0), (3, 3, 1), (3, 3, 2)\}$
405) $R_{6,-,-;3}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 0), (0, 3, 1), (2, 0, 0), (2, 1, 1), (2, 2, 1), (2, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 1), (1, 0, 1), (1, 0, 2), (1, 1, 0), (1, 1, 1), (1, 2, 1), (2, 0, 1), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
406) $R_{6,-,-;4}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 0), (0, 3, 0), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 0, 0), (2, 1, 0), (2, 2, 1), (2, 3, 1), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (2, 0, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_3 = \{(0, 1, 1), (0, 2, 0), (1, 1, 0), (1, 2, 1), (2, 0, 0), (2, 3, 1), (3, 0, 1), (3, 3, 0)\}$
407) $R_{6,-,-;5}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 0), (0, 3, 1), (1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 1), (2, 0, 0), (2, 1, 1), (2, 2, 1), (2, 3, 0), (3, 0, 1), (3, 1, 1), (3, 2, 1), (3, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 1), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 1), (2, 0, 1), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 1), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
408) $R_{6,-,-;6}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 1), (0, 2, 0), (0, 3, 0), (1, 0, 0), (1, 1, 0), (1, 2, 1), (1, 3, 1), (2, 0, 0), (2, 1, 0), (2, 2, 1), (2, 3, 1), (3, 0, 1), (3, 1, 1), (3, 2, 0), (3, 3, 0)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (2, 0, 0), (2, 2, 1), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (1, 1, 1), (1, 2, 0), (2, 0, 1), (2, 3, 0), (3, 0, 0), (3, 3, 1)\}$
409) $R_{6,-,-;7}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 1), (3, 0, 1), (5, 0, 1), (5, 1, 0), (5, 1, 1), (7, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (3, 1, 1), (4, 0, 0), (5, 0, 0), (6, 0, 0), (6, 0, 1), (6, 1, 0), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (2, 1, 1), (3, 1, 1), (4, 0, 0), (5, 0, 0), (5, 0, 1), (5, 1, 1), (6, 0, 1), (7, 0, 1)\}$
410) $R_{6,-,-;8}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(3, 0, 1), (3, 1, 1), (5, 0, 1), (5, 1, 0), (7, 1, 0), (7, 1, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 0, 0), (3, 0, 1), (4, 0, 0), (5, 0, 0), (6, 0, 0), (6, 1, 0), (6, 1, 1), (7, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 0, 1), (2, 1, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (4, 1, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (6, 0, 1), (7, 0, 1)\}$
411) $R_{6,-,-;9}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (5, 1, 1), (7, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 1, 1), (4, 0, 0), (5, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 1), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (2, 0, 1), (3, 1, 1), (4, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (6, 1, 1), (7, 0, 1)\}$
412) $R_{6,-,-;10}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (3, 0, 1), (3, 1, 1), (7, 1, 0), (7, 1, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (2, 0, 0), (2, 1, 0), (2, 1, 1), (3, 0, 1), (4, 0, 0), (5, 0, 1), (6, 0, 0), (7, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (2, 0, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (4, 1, 1), (5, 0, 1), (6, 1, 1), (7, 0, 1)\}$
413) $R_{6,-,-;11}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (3, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (2, 1, 1), (3, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0), (2, 1, 1), (3, 0, 1)\}$
414) $R_{6,-,-;12}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 1, 0), (1, 1, 1), (3, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 0, 0), (2, 0, 1), (2, 1, 1), (3, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (2, 0, 1), (3, 0, 1)\}$
415) $R_{6,-,-;13}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 1, 0), (3, 1, 0), (3, 1, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 0, 0), (3, 0, 1), (3, 1, 0), (3, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (2, 0, 1), (3, 0, 1)\}$
416) $R_{6,-,-;14}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (3, 0, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (7, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 1, 0), (3, 1, 1), (4, 0, 0), (5, 0, 0), (6, 1, 1), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0), (2, 1, 1), (3, 1, 1), (4, 0, 0), (5, 1, 0), (6, 0, 1), (7, 0, 1)\}$
417) $R_{6,-,-;15}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 1, 1), (3, 0, 1), (3, 1, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (7, 1, 0), (7, 1, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 0, 1), (2, 1, 0), (2, 1, 1), (3, 0, 1), (4, 0, 0), (5, 0, 0), (6, 0, 1), (7, 0, 1), (7, 1, 0), (7, 1, 1)\}$ $V_3 = \{(0, 1, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 1, 1), (3, 1, 1), (4, 0, 0), (4, 1, 0), (4, 1, 1), (5, 0, 1), (6, 0, 1), (7, 0, 1)\}$
418) $R_{6,-,-;16}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 1), (3, 1, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (7, 0, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 1, 1), (4, 0, 0), (5, 0, 1), (6, 0, 0), (6, 0, 1), (6, 1, 1), (7, 1, 0)\}$ $V_3 = \{(0, 1, 0), (1, 0, 0), (1, 0, 1), (1, 1, 1), (2, 0, 1), (3, 1, 1), (4, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (6, 1, 1), (7, 0, 1)\}$
419) $R_{6,-,-;17}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 0, 1), (3, 1, 0)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 0, 1), (3, 0, 1)\}$ $V_3 = \{(0, 1, 1), (1, 1, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 1, 1)\}$
420) $R_{6,-,-;18}$	$p_1 = 4, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 1, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (2, 0, 1), (3, 0, 1)\}$ $V_3 = \{(0, 1, 1), (1, 1, 1), (2, 0, 0), (2, 0, 1), (2, 1, 0), (3, 1, 1)\}$
421) $R_{6,-,-;19}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (1, 1, 1), (3, 1, 0), (5, 1, 1), (7, 0, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 1), (2, 1, 1), (3, 0, 0), (3, 0, 1), (3, 1, 1), (4, 0, 0), (5, 0, 0), (6, 1, 0), (7, 0, 0), (7, 0, 1), (7, 1, 0)\}$ $V_3 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (1, 1, 0), (2, 1, 1), (3, 1, 1), (4, 0, 0), (4, 0, 1), (4, 1, 1), (5, 0, 0), (6, 0, 1), (7, 0, 1)\}$
422) $R_{6,-,-;20}$	$p_1 = 8, p_2 = 2, p_3 = 2$ $V_1 = \{(1, 1, 1), (3, 1, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (7, 0, 1)\}$ $V_2 = \{(0, 0, 1), (1, 0, 0), (2, 1, 0), (3, 0, 0), (3, 0, 1), (3, 1, 1), (4, 0, 0), (5, 0, 1), (6, 1, 1), (7, 0, 0), (7, 0, 1), (7, 1, 0)\}$

Table 2 (cont.)

Σ	Edges
	$(4, 2, 0), (4, 2, 1), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 1), (5, 2, 0), (6, 1, 1), (6, 2, 1), (7, 1, 1), (7, 2, 1)$
473) $R_{8,-,-;19}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 1, 1), (1, 3, 0), (3, 1, 0), (3, 3, 1), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 1), (7, 0, 1), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_2 = \{(1, 0, 0), (1, 0, 1), (1, 2, 0), (1, 2, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 1, 1), (3, 2, 1), (3, 3, 0), (4, 0, 0), (4, 0, 1), (4, 2, 0), (4, 2, 1), (6, 0, 1), (6, 1, 1), (6, 2, 0), (6, 3, 0), (7, 0, 0), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 1, 1), (2, 2, 1), (3, 0, 1), (3, 3, 1), (4, 0, 0), (4, 2, 0), (4, 2, 1), (4, 3, 1), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 3, 0), (6, 0, 1), (6, 3, 1), (7, 1, 1), (7, 2, 1)\}$
474) $R_{8,-,-;20}$	$p_1 = 4, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (3, 0, 1), (3, 2, 1), (3, 3, 0), (3, 3, 1)\}$ $V_2 = \{(2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 1), (3, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 1, 1), (2, 2, 1), (3, 1, 1), (3, 2, 1)\}$
475) $R_{8,-,-;21}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (0, 3, 0), (1, 1, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 2, 1), (2, 3, 0), (3, 0, 1), (3, 2, 1), (3, 3, 0), (3, 3, 1), (4, 0, 1), (4, 1, 0), (4, 2, 1), (4, 3, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 1), (6, 0, 1), (6, 1, 0), (6, 2, 1), (6, 3, 0), (7, 1, 0), (7, 3, 1)\}$ $V_2 = \{(2, 0, 1), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 1), (3, 2, 1), (3, 3, 0), (4, 0, 0), (4, 0, 1), (4, 2, 0), (4, 2, 1), (5, 0, 0), (5, 0, 1), (5, 2, 0), (5, 2, 1), (6, 0, 1), (6, 1, 0), (6, 2, 0), (6, 3, 1), (7, 0, 0), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 3, 0), (2, 0, 1), (2, 3, 1), (3, 0, 1), (3, 3, 1), (4, 0, 0), (4, 2, 0), (4, 2, 1), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (6, 1, 1), (6, 2, 1), (7, 1, 1), (7, 2, 1)\}$
476) $R_{8,-,-;22}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(0, 0, 1), (0, 1, 0), (0, 2, 1), (0, 3, 0), (1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 0), (2, 0, 1), (2, 1, 0), (2, 2, 1), (2, 3, 0), (3, 3, 0), (3, 3, 1), (4, 0, 1), (4, 1, 0), (4, 2, 1), (4, 3, 0), (5, 1, 0), (5, 1, 1), (6, 0, 1), (6, 1, 0), (6, 2, 1), (6, 3, 0), (7, 0, 1), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 1), (1, 0, 1), (1, 2, 1), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 2, 1), (4, 0, 0), (4, 2, 0), (5, 0, 0), (5, 2, 0), (6, 0, 1), (6, 2, 0), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1), (2, 1, 1), (2, 3, 1), (3, 2, 1), (3, 3, 1), (4, 0, 0), (4, 1, 0), (4, 2, 0), (4, 2, 1), (4, 3, 0), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (5, 3, 0), (6, 0, 1), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$
477) $R_{8,-,-;23}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 1), (3, 0, 1), (3, 2, 1), (3, 3, 0), (3, 3, 1), (5, 1, 1), (5, 3, 0), (7, 1, 0), (7, 3, 1)\}$ $V_2 = \{(0, 1, 0), (0, 1, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 2, 1), (1, 3, 0), (1, 3, 1), (2, 0, 1), (2, 1, 0), (2, 2, 0), (2, 3, 1), (3, 0, 0), (3, 1, 1), (3, 2, 1), (3, 3, 0), (4, 0, 0), (4, 0, 1), (4, 1, 0), (4, 1, 1), (4, 2, 0), (4, 2, 1), (4, 3, 0), (4, 3, 1), (5, 1, 0), (5, 1, 1), (5, 3, 0), (5, 3, 1), (6, 0, 1), (6, 1, 1), (6, 2, 0), (6, 3, 0), (7, 0, 0), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 1), (1, 2, 0), (2, 0, 0), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 2, 1), (2, 3, 0), (3, 0, 0), (3, 0, 1), (3, 1, 0), (3, 2, 0), (3, 3, 0), (3, 3, 1), (4, 0, 0), (4, 2, 0), (4, 2, 1), (4, 3, 1), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 3, 0), (6, 0, 0), (6, 0, 1), (6, 1, 0), (6, 2, 0), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 1, 0), (7, 1, 1), (7, 2, 0), (7, 2, 1), (7, 3, 0)\}$
478) $R_{8,-,-;24}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 0), (3, 0, 1), (3, 1, 0), (3, 2, 1), (3, 3, 1), (5, 1, 0), (5, 1, 1), (7, 3, 0), (7, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 1, 0), (0, 1, 1), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0), (1, 3, 1), (2, 0, 1), (2, 2, 0), (2, 3, 0), (2, 3, 1), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 2, 1), (4, 0, 0), (4, 1, 0), (4, 1, 1), (4, 2, 0), (4, 3, 0), (4, 3, 1), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 1), (5, 3, 0), (5, 3, 1), (6, 0, 1), (6, 1, 0), (6, 1, 1), (6, 2, 0), (6, 3, 0), (7, 0, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 1, 1), (1, 2, 0), (1, 3, 0), (2, 0, 0), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (2, 3, 0), (3, 0, 0), (3, 1, 0), (3, 2, 0), (3, 2, 1), (3, 3, 0), (3, 3, 1), (4, 0, 0), (4, 1, 0), (4, 2, 0), (4, 2, 1), (4, 3, 0), (4, 3, 1), (5, 0, 1), (5, 1, 1), (6, 0, 0), (6, 1, 0), (6, 2, 0), (6, 2, 1), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 0, 1), (7, 1, 0), (7, 1, 1), (7, 2, 0), (7, 3, 0)\}$
479) $R_{8,-,-;25}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 0, 1), (1, 1, 1), (1, 2, 1), (1, 3, 0), (3, 3, 0), (3, 3, 1), (5, 1, 0), (5, 1, 1), (7, 0, 1), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_2 = \{(0, 0, 1), (0, 2, 1), (1, 0, 1), (1, 2, 1), (2, 0, 1), (2, 1, 0), (2, 1, 1), (2, 2, 0), (3, 0, 0), (3, 1, 0), (3, 1, 1), (3, 2, 1), (4, 0, 0), (4, 2, 0), (5, 0, 0), (5, 2, 0), (6, 0, 1), (6, 2, 0), (6, 3, 0), (6, 3, 1), (7, 0, 0), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_3 = \{(0, 2, 1), (0, 3, 1), (1, 0, 1), (1, 1, 1), (2, 2, 1), (2, 3, 1), (3, 2, 1), (3, 3, 1), (4, 0, 0), (4, 1, 0), (4, 2, 0), (4, 2, 1), (4, 3, 0), (4, 3, 1), (5, 0, 0), (5, 0, 1), (5, 1, 0), (5, 1, 1), (5, 2, 0), (5, 3, 0), (6, 0, 1), (6, 1, 1), (7, 0, 1), (7, 1, 1)\}$
480) $R_{8,-,-;26}$	$p_1 = 8, p_2 = 4, p_3 = 2$ $V_1 = \{(1, 0, 0), (1, 0, 1), (1, 1, 0), (1, 2, 0), (1, 2, 1), (1, 3, 1), (3, 1, 0), (3, 3, 1), (5, 0, 0), (5, 2, 0), (5, 3, 0), (5, 3, 1), (7, 0, 1), (7, 2, 1), (7, 3, 0), (7, 3, 1)\}$ $V_2 = \{(2, 0, 0), (2, 1, 0), (2, 2, 1), (2, 3, 1), (3, 0, 0), (3, 1, 1), (3, 2, 1), (3, 3, 0), (4, 0, 0), (4, 0, 1), (4, 2, 0), (4, 2, 1), (5, 0, 0), (5, 0, 1), (5, 2, 0), (5, 2, 1), (6, 0, 0), (6, 1, 1), (6, 2, 1), (6, 3, 0), (7, 0, 0), (7, 1, 0), (7, 2, 1), (7, 3, 1)\}$ $V_3 = \{(0, 1, 0), (0, 2, 1), (0, 3, 0), (0, 3, 1), (1, 0, 0), (1, 2, 0), (1, 2, 1), (1, 3, 1), (2, 0, 1), (2, 3, 1), (3, 0, 1), (3, 3, 1), (4, 0, 0), (4, 2, 0), (4, 2, 1), (4, 3, 1), (5, 1, 0), (5, 2, 1), (5, 3, 0), (5, 3, 1), (6, 1, 1), (6, 2, 1), (7, 1, 1), (7, 2, 1)\}$

REFERENCES

1. Trofimov V.I. Symmetrical extensions of graphs and some other topics in graph theory related with group theory. *Proc. Steklov Inst. Math.*, 2012, vol. 279, suppl. 1, pp. S107–S112. <https://doi.org/10.1134/S0081543812090088>
2. Neganova E.A., Trofimov V.I. Symmetrical extensions of graphs. *Izv. Math.*, 2014, vol. 78, no. 4, pp. 809–835. <https://doi.org/10.1070/IM2014v078n04ABEH002707>
3. Trofimov V.I. The finiteness of the number of symmetrical 2-extensions of the d -dimensional grid and similar graphs. *Proc. Steklov Inst. Math.*, 2014, vol. 285, suppl. 1, pp. S169–S182. <https://doi.org/10.1134/S0081543814050198>
4. Konvalchik E.A., Kostousov K.V. Symmetrical 2-extensions of the 2-dimensional grid. I. *Trudy Inst. Mat. Mekh. UrO RAN*, 2016, vol. 22, no. 1, pp. 159–179 (in Russian).
5. Konvalchik E.A., Kostousov K.V. Symmetrical 2-extensions of the 2-dimensional grid. II. *Trudy Inst. Mat. Mekh. UrO RAN*, 2017, vol. 23, no. 4, pp. 192–211 (in Russian). <https://doi.org/10.21538/0134-4889-2017-23-4-192-211>
6. Kostousov K.V. Symmetrical 2-extensions of the 3-dimensional grid. I. *The Art Discrete and Applied Mathematics.*, 2021, vol. 4, no. 2, art. no. P2.04, 155 p. <https://doi.org/10.26493/2590-9770.1353.c0e>

7. Eick B., Horn M, Nickel W. GAP package Polycyclic. Ver. 2.11: [e-resource]. 2013. <https://www.gap-system.org/Packages/polycyclic.html>
8. Eick B., Gahler F., Nickel W. GAP package Cryst — Computing with crystallographic groups. Ver. 4.1: [e-resource]. 2013. <https://www.gap-system.org/Packages/cryst.html>
9. GAP — Groups, Algorithms, Programming - a System for Computational Discrete Algebra. Ver. 4.5.7: [e-resource]. 2012. <https://www.gap-system.org>
10. Vasev P.A. Vizualization system for 2-extensions of 3-dimensional grid. 2016. <http://viewlang.ru/3dgrid>

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