

A Variant of Stechkin's Problem on the Best Approximation of a Fractional Order Differentiation Operator on the Axis

V. V. Arestov^{1,2}

Received June 19, 2024; revised September 17, 2024; accepted September 23, 2024

Abstract—A solution is given to Stechkin's problem on the best approximation on the real axis of differentiation operators of fractional (more precisely, real) order k in the space L_2 by bounded linear operators from the space L to the space L_2 on the class of functions whose fractional derivative of order n , $0 \leq k < n$, is bounded in the space L_2 . An upper estimate is obtained for the best constant in the corresponding Kolmogorov inequality. It is shown that the well-known Stechkin lower estimate for the value of the problem of approximating the differentiation operator via the best constant in the Kolmogorov inequality is strict in this case; in other words, Stechkin's problem and the Kolmogorov inequality are not consistent.

Keywords: fractional order differentiation operator, Stechkin's problem, Kolmogorov inequality, Carlson inequality.

DOI: [10.1134/S0081543824070022](https://doi.org/10.1134/S0081543824070022)

¹Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia

²Ural Federal University, Yekaterinburg, 620000 Russia
e-mail: vitalii.arestov@urfu.ru