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Lattice Characterizations of *p*-Soluble and *p*-Supersoluble Finite Groups

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Abstract—Let G be a finite group, and let $\mathcal{L}(G)$ be the lattice of all subgroups of G. A subgroup M of G is called *modular* in G if M is a modular element (in the Kurosh sense) of the lattice $\mathcal{L}(G)$, i.e., if (1) $\langle X, M\cap Z\rangle = \langle X, M\rangle \cap Z$ for all $X \leq G, Z \leq G$ such that $X \leq Z$, and (2) $\langle M, Y\cap Z\rangle = \langle M, Y\rangle \cap Z$ for all $Y \leq G, Z \leq G$ such that $M \leq Z$. If A is a subgroup of G, then A_{mG} is the subgroup of A generated by all its subgroups that are modular in G. We say that a subgroup A is N-*modular* in G ($N \leq G$) if, for some modular subgroup T of G containing A, N *avoids the pair* (T, A_{mG}), i.e., $N \cap T = N \cap A_{mG}$. Using these notions, we give new characterizations of p-soluble and p-supersoluble finite groups.

Keywords: finite group, p-soluble group, p-supersoluble group, modular subgroup, N-modular subgroup.

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