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Strict Suns Composed of Planes

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Abstract—A set M is a strict sun if, for each $x \notin M$, the set $P_M x$ of best approximants from M for x is nonempty and each point $y \in P_M x$ is a nearest point from M for each point z from the ray emanating from y and passing through x. Strict suns are sometimes called Kolmogorov sets, because they satisfy the Kolmogorov criterion for best approximation. We study the structural properties of strict suns composed of a a finite number of planes (affine spaces, which may possibly degenerate to points). We always assume that the union of planes $M := \bigcup L_i$ is irreducible, i.e., no plane in this union contains another plane from the union. We show that if an irreducible finite union of planes $M := \bigcup_{i=1}^N L_i$ is a strict sun in a normed space, then M consists of a single plane. In this result, the strict sun cannot be replaced by a sun. A stronger local analog of this result is proved in the space ℓ_n^∞. Namely, we show that if $M := \bigcup_{i=1}^N L_i$ is an irreducible union of planes in ℓ_n^∞, Π is a bar (intersection of extreme hyperplanes), and $M \cap \Pi \neq \varnothing$, then $M' := M \cap \Pi$ is a strict sun in ℓ_n^∞ if and only if M' is convex, i.e., M' is the intersection of some plane L_i with the bar Π. As a corollary, if $M := \bigcup_{i=1}^N L_i$ is a local strict sun in ℓ_n^∞, then M consists of a single plane. Similar results are also established for sets $M := \bigcup_{i=1}^N L_i$ with continuous metric projection in ℓ_n^∞. The present paper continues and develops the previous studies on approximation by Chebyshev sets composed of planes begun by A.R. Alimov and I.G. Tsar'kov in linear normed and asymmetrically normed spaces and the results of I.G. Tsar'kov on sets with a piecewise continuous metric projection.

Keywords: best approximation, union of planes, sun, strict sun, discretization.

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