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## The Value and Optimal Strategies in a Positional Differential Game for a Neutral-Type System

M. I. Gomoyunov<sup>1,2,\*</sup> and N. Yu. Lukoyanov<sup>1,\*\*</sup>

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Abstract—On a finite time interval, a differential game for the minimax–maximin of a given cost functional is considered. In this game, the motion of a conflict-controlled dynamical system is described by functional differential equations of neutral type in Hale's form. Under assumptions more general than those considered previously, a theorem on the existence of the value and saddle point of the game in classes of players' closed-loop control strategies with memory of the motion history is proved. The proof involves the technique of the corresponding path-dependent Hamilton–Jacobi equations with coinvariant derivatives and the theory of minimax (generalized) solutions of such equations. In order to construct optimal strategies, which constitute a saddle point of the game, a recent result on the existence and uniqueness of a suitable minimax solution and a special Lyapunov–Krasovskii functional are used.

**Keywords:** differential game, neutral-type equation, game value, optimal strategies, pathdependent Hamilton–Jacobi equation, coinvariant derivatives, minimax solution.

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<sup>2</sup>Udmurt State University, Izhevsk, 426034 Russia

e-mail: \*m.i.gomoyunov@gmail.com,\*\*nyul@imm.uran.ru

<sup>&</sup>lt;sup>1</sup>Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia