Integro-Differential Equations of Gerasimov Type with Sectorial Operators

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Abstract—The issues of existence and uniqueness of a solution to the Cauchy problem are studied for a linear equation in a Banach space with a closed operator at the unknown function that is resolved with respect to a first-order integro-differential operator of Gerasimov type. The properties of resolving families of operators of the homogeneous equations are investigated. It is shown that sectoriality, i.e., belonging to the class of operators $\mathcal{A}_K$ introduced here, is a necessary and sufficient condition for the existence of an analytical resolving family of operators in a sector. A theorem on the perturbation of operators of the class $\mathcal{A}_K$ is obtained, and two versions of the theorem on the existence and uniqueness of a solution to a linear inhomogeneous equation are proved. Abstract results are used to study initial–boundary value problems for an equation with the Prabhakar time derivative and for a system of partial differential equations with Gerasimov–Caputo time derivatives of different orders.

Keywords: integro-differential equation, Gerasimov–Caputo derivative, Cauchy problem, sectorial operator, resolving family of operators, initial–boundary value problem.

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