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A Stable Solution of a Nonuniformly Perturbed Quadratic Minimization Problem by the Extragradient Method with Step Size Separated from Zero

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Abstract—A quadratic minimization problem is considered in Hilbert spaces under constraints given by a linear operator equation and a convex quadratic inequality. The main feature of the problem statement is that the practically available approximations to the exact linear operators specifying the criterion and the constraints converge to them only strongly pointwise rather than in the uniform operator norm, which makes it impossible to justify the use of the classical regularization methods. We propose a regularization method that is applicable in the presence of error estimates for approximate operators in pairs of other operator norms, which are weaker than the original ones. For each of the operators, the pair of corresponding weakened operator norms is obtained by strengthening the norm in the domain of the operator and weakening the norm in its range. The weakening of operator norms usually makes it possible to estimate errors in operators where this was fundamentally impossible in the original norms, for example, in the finite-dimensional approximation of a noncompact operator. From the original optimization formulation, a transition is made to the problem of finding a saddle point of the Lagrange function. The proposed numerical method for finding a saddle point is an iterative regularized extragradient two-stage procedure. At the first stage of each iteration, an approximation to the optimal value of the criterion is refined; at the second stage, the approximate solution with respect to the main variable is refined. Compared to the methods previously developed by the authors and working under similar information conditions, this method is preferable for practical implementation, since it does not require the gradient step size to converge to zero. The main result of the work is the proof of the strong convergence of the approximations generated by the method to one of the exact solutions to the original problem in the norm of the original space.

Keywords: quadratic minimization problem, approximate data, numerical solution, ill-posed problem, regularization, extragradient method, Lagrange function, saddle point.

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