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The Method of Comparison with a Model Equation in the Study of Inclusions in Vector Metric Spaces

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Abstract—For a given multivalued mapping $F: X \rightrightarrows Y$ and a given element $\tilde{y} \in Y$, the existence of a solution $x \in X$ to the inclusion $F(x) \ni \tilde{y}$ and its estimates are studied. The sets X and Y are endowed with vector-valued metrics $\mathcal{P}_X^{E_+}$ and $\mathcal{P}_Y^{M_+}$, whose values belong to cones E_+ and M_+ of a Banach space E and a linear topological space M, respectively. The inclusion is compared with a "model" equation $f(t) = 0$, where $f: E_+ \to M$. It is assumed that f can be written as $f(t) \equiv g(t,t)$, where the mapping $g: E_+ \times E_+ \to M$ orderly covers the set $\{0\} \subset M$ with respect to the first argument and is antitone with respect to the second argument and $-g(0,0) \in M_+$. It is shown that, in this case, the equation $f(t) = 0$ has a solution $t^* \in E_+$. Further, conditions on the connection between $f(0)$ and $F(x_0)$ and between the increments of $f(t)$ for $t \in [0, t^*]$ and the increments of $F(x)$ for all x in the ball of radius t^* centered at x_0 for some x_0 are formulated, and it is shown that the inclusion has a solution in the ball under these conditions. The results on the operator inclusion obtained in the paper are applied to studying an integral inclusion.

Keywords: operator inclusion, existence and estimates of solutions, integral inclusion, vector metric space.

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