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Reidemeister Torsion for Vector Bundles on $\mathbb{P}^1_{\mathbb{Z}}$

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Abstract—We consider vector bundles of rank 2 with trivial generic fiber on the projective line over \mathbb{Z} . For such bundles, a new invariant is constructed — the Reidemeister torsion, which is an analog of the classical Reidemeister torsion from topology. For vector bundles of rank 2 with trivial generic fiber and jumps of height 1, that is, for the bundles that are isomorphic to \mathcal{O}^2 in the fiber over \mathbb{Q} and are isomorphic to \mathcal{O}^2 or $\mathcal{O}(-1) \oplus \mathcal{O}(1)$ over each closed point of Spec(\mathbb{Z}), we calculate this invariant and show that it, together with the discriminant of the bundle, completely determines such a bundle.

Keywords: vector bundle, arithmetic surface, projective line, torsion.

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