

# Reidemeister Torsion for Vector Bundles on $\mathbb{P}_{\mathbb{Z}}^1$

V. M. Polyakov<sup>1</sup>

Received November 29, 2023; revised December 19, 2023; accepted December 25, 2023

**Abstract**—We consider vector bundles of rank 2 with trivial generic fiber on the projective line over  $\mathbb{Z}$ . For such bundles, a new invariant is constructed—the Reidemeister torsion, which is an analog of the classical Reidemeister torsion from topology. For vector bundles of rank 2 with trivial generic fiber and jumps of height 1, that is, for the bundles that are isomorphic to  $\mathcal{O}^2$  in the fiber over  $\mathbb{Q}$  and are isomorphic to  $\mathcal{O}^2$  or  $\mathcal{O}(-1) \oplus \mathcal{O}(1)$  over each closed point of  $\text{Spec}(\mathbb{Z})$ , we calculate this invariant and show that it, together with the discriminant of the bundle, completely determines such a bundle.

**Keywords:** vector bundle, arithmetic surface, projective line, torsion.

**DOI:** 10.1134/S008154382403012X

---

<sup>1</sup>St. Petersburg Department of the Steklov Mathematical Institute of the Russian Academy of Sciences, St. Petersburg, 191023 Russia  
e-mail: polyakov@pdmi.ras.ru