ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2023, Vol. 323, Suppl. 1, pp. S53–S55.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2023, Vol. 29, No. 4, pp. 279–282.

On Graphs in Which the Neighborhoods of Vertices Are Edge-Regular Graphs without 3-Claws

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Received August 22, 2023; revised September 12, 2023; accepted September 18, 2023

Abstract—The triangle-free Krein graph $\mathrm{Kre}(r)$ is strongly regular with parameters $((r^2+3r)^2, r^3+3r^2+r, 0, r^2+r)$. The existence of such graphs is known only for $r=1$ (the complement of the Clebsch graph) and $r=2$ (the Higman–Sims graph). A. L. Gavrilyuk and A. A. Makhnev proved that the graph $\mathrm{Kre}(3)$ does not exist. Later Makhnev proved that the graph $\mathrm{Kre}(4)$ does not exist. The graph $\mathrm{Kre}(r)$ is the only strongly regular triangle-free graph in which the antineighborhood of a vertex $\mathrm{Kre}(r)'$ is strongly regular. The graph $\mathrm{Kre}(r)'$ has parameters $((r^2+2r-1)(r^2+3r+1), r^3+2r^2, 0, r^2)$. This work clarifies Makhnev's result on graphs in which the neighborhoods of vertices are strongly regular graphs without 3-cocliques. As a consequence, it is proved that the graph $\mathrm{Kre}(r)$ exists if and only if the graph $\mathrm{Kre}(r)'$ exists and is the complement of the block graph of a quasi-symmetric 2-design.

Keywords: distance-regular graph, strongly regular graph.

DOI: 10.1134/S0081543823060044

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