

On Submodularity and $K\mathfrak{F}$ -Subnormality in Finite Groups

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Abstract—Let \mathfrak{F} be a formation, and let G be a finite group. A subgroup H of G is called $K\mathfrak{F}$ -subnormal (submodular) in G if there is a subgroup chain $H = H_0 \leq H_1 \leq \dots \leq H_{n-1} \leq H_n = G$ such that, for every i either H_i is normal in H_{i+1} or $H_{i+1}^{\mathfrak{F}} \leq H_i$ (H_i is a modular subgroup of H_{i+1} , respectively). We prove that, in a group, a primary subgroup is submodular if and only if it is $K\mathfrak{U}_1$ -subnormal. Here \mathfrak{U}_1 is a formation of all supersolvable groups of square-free exponent. Moreover, for a solvable subgroup-closed formation \mathfrak{F} , every solvable $K\mathfrak{F}$ -subnormal subgroup of a group G is contained in the solvable radical of G . We also obtain a series of applications of these results to the investigation of groups factorized by $K\mathfrak{F}$ -subnormal and submodular subgroups.

Keywords: finite group, subnormal subgroup, submodular subgroup.

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