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Periodic Groups with One Finite Nontrivial Sylow 2-Subgroup

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Abstract—The following results are proved. Let d be a natural number, and let G be a group of finite even exponent such that each of its finite subgroups is contained in a subgroup isomorphic to the direct product of m dihedral groups, where $m \leq d$. Then G is finite (and isomorphic to the direct product of at most d dihedral groups). Next, suppose that G is a periodic group and p is an odd prime. If every finite subgroup of G is contained in a subgroup isomorphic to the direct product $D_1 \times D_2$, where D_i is a dihedral group of order $2p^{r_i}$ with natural r_i , i = 1, 2, then $G = M_1 \times M_2$, where $M_i = \langle H_i, t \rangle$, t_i is an element of order 2, H_i is a locally cyclic p-group, and $h^{t_i} = h^{-1}$ for every $h \in H_i$, i = 1, 2. Now, suppose that d is a natural number and G is a solvable periodic group such that every of its finite subgroups is contained in a subgroup isomorphic to the direct product of an abelian normal subgroup by an elementary abelian 2-subgroup of order at most 2^{2d} .

Keywords: periodic group, exponent, Sylow 2-subgroup, dihedral group, direct product, saturating set.

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