ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2023, Vol. 323, Suppl. 1, pp. S146–S154.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2023, Vol. 29, No. 4, pp. 130–139.

On Constants in the Bernstein–Szegő Inequality for the Weyl Derivative of Order Less Than Unity of Trigonometric Polynomials and Entire Functions of Exponential Type in the Uniform Norm

A. O. Leont'eva[1]

Received July 3, 2023; revised August 8, 2023; accepted August 14, 2023

Abstract—The Weyl derivative (fractional derivative) $f_n^{(\alpha)}$ of real nonnegative order α is considered on the set $\mathscr{T}_n$ of trigonometric polynomials f_n of order n with complex coefficients. The constant in the Bernstein–Szegő inequality $\|f_n^{(\alpha)}\cos\theta + \tilde{f}_n^{(\alpha)}\sin\theta\| \le B_n(\alpha,\theta)\|f_n\|$ in the uniform norm is studied. This inequality has been well studied for $\alpha \ge 1$: G. T. Sokolov proved in 1935 that it holds with the constant n^α for all $\theta \in \mathbb{R}$. For $0 < \alpha < 1$, there is much less information about $B_n(\alpha,\theta)$. In this paper, for $0 < \alpha < 1$ and $\theta \in \mathbb{R}$, we establish the limit relation $\lim_{n\to\infty} B_n(\alpha,\theta)/n^\alpha = \mathcal{B}(\alpha,\theta)$, where $\mathcal{B}(\alpha,\theta)$ is the sharp constant in the similar inequality for entire functions of exponential type at most 1 that are bounded on the real line. The value $\theta = -\pi\alpha/2$ corresponds to the Riesz derivative, which is an important particular case of the Weyl–Szegő operator. In this case, we derive exact asymptotics for the quantity $B_n(\alpha) = B_n(\alpha, -\pi\alpha/2)$ as $n \to \infty$.

Keywords: trigonometric polynomials, entire functions of exponential type, Weyl–Szegő operator, Riesz derivative, Bernstein inequality, uniform norm.

DOI: 10.1134/S0081543823060123

[1]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
e-mail: lao-imm@yandex.ru