

# Closed Mappings and Construction of Extension Models

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Received April 13, 2023; revised May 12, 2023; accepted May 15, 2023

**Abstract**—The problem of reachability in a topological space is studied under constraints of asymptotic nature arising from weakening the requirement that the image of a solution belong to a given set. The attraction set that arises in this case in the topological space is a regularization of certain kind for the image of the preimage of the mentioned set (the image and the preimage are defined for generally different mappings). When constructing natural compact extensions of the reachability problem with constraints of asymptotic nature generated by a family of neighborhoods of a fixed set, the case was studied earlier where the topological space in which the results of one or another choice of solution are realized satisfies the axiom  $T_2$ . In the present paper, for a number of statements related to compact extensions, it is possible to use for this purpose a  $T_1$  space, which seems to be quite important from a theoretical point of view, since it is possible to find out the exact role of the axiom  $T_2$  in questions related to correct extensions of reachability problems. We study extension models using ultrafilters of a broadly understood measurable space with detailing of the main elements in the case of a reachability problem in the space of functionals with the topology of a Tychonoff power of the real line with the usual  $|\cdot|$ -topology. The general constructions of extension models are illustrated by an example of a nonlinear control problem with state constraints.

**Keywords:** attraction set, extension model, ultrafilter.

**DOI:** 10.1134/S0081543823060056

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