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On Operator Inclusions in Spaces with Vector-Valued Metrics

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Abstract—We consider an inclusion $\widetilde{y} \in F(x)$ with a multivalued mapping acting in spaces with vector-valued metrics whose values are elements of cones in Banach spaces and can be infinite. A statement about the existence of a solution $x \in X$ and an estimate of its deviation from a given element $x_0 \in X$ in a vector-valued metric are obtained. This result extends the known theorems on similar operator equations and inclusions in metric spaces and in the spaces with n-dimensional metric to a more general case and, applied to particular classes of functional equations and inclusions, allows to get less restrictive, compared to known, solvability conditions as well as more precise estimates of solutions. We apply this result to the integral inclusion

$$\widetilde{y}(t) \in f\Big(t, \int_a^b \varkappa(t,s)x(s)\,ds, x(t)\Big), \quad t \in [a,b],$$

where the function $\widetilde{y}$ is measurable, the mapping f satisfies the Carathéodory conditions, and the solution x is required to be only measurable (the integrability of x is not assumed).

Keywords: space with vector-valued metric, multivalued mapping, vector metric regularity, Lipschitz property with operator coefficient, operator inclusion, integral inclusion.

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