

# Adaptive Subgradient Methods for Mathematical Programming Problems with Quasiconvex Functions

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**Abstract**—The paper is devoted to subgradient methods with switching between productive and nonproductive steps for problems of minimization of quasiconvex functions under functional inequality constraints. For the problem of minimizing a convex function with quasiconvex inequality constraints, a result is obtained on the convergence of the subgradient method with an adaptive stopping rule. Further, based on an analog of a sharp minimum for nonlinear problems with inequality constraints, results are obtained on the geometric convergence of restarted versions of subgradient methods. Such results are considered separately in the case of a convex objective function and quasiconvex inequality constraints, as well as in the case of a quasiconvex objective function and convex inequality constraints. The convexity may allow to additionally suggest adaptive stopping rules for auxiliary methods, which guarantee that an acceptable solution quality is achieved. The results of computational experiments are presented, showing the advantages of using such stopping rules.

**Keywords:** subgradient method, quasiconvex function, sharp minimum, restarts, adaptive method.

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