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On Intersections of Nilpotent Subgroups in Finite Groups with Simple Socle from the "Atlas of Finite Groups"

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Abstract—Earlier, the author described up to conjugacy all pairs (A, B) of nilpotent subgroups of a finite group G with socle $L_2(q)$ for which $A \cap B^g \neq 1$ for any element of G. A similar description was obtained by the author later for primary subgroups A and B of a finite group G with socle $L_n(2^m)$. In this paper, we describe up to conjugacy all pairs (A, B) of nilpotent subgroups of a finite group G with simple socle from the "Atlas of Finite Groups" for which $A \cap B^g \neq 1$ for any element q of G. The results obtained in the considered cases confirm the hypothesis (Problem 15.40 from the "Kourovka Notebook") that a finite simple nonabelian group G for any nilpotent subgroups N contains an element g such that $N \cap N^g = 1$.

Keywords: finite group, nilpotent subgroup, intersection of subgroups, Fitting subgroup.

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