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A Study of New Methods for Localizing Discontinuity Lines on Extended Correctness Classes

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Abstract—We consider the ill-posed problem of finding the position of the discontinuity lines of a function of two variables. It is assumed that the function is smooth outside the lines of discontinuity but has a discontinuity of the first kind on the line. At each node of a uniform grid with step τ, the mean values of the perturbed function on a square with side τ are known. The perturbed function approximates the exact function in the space $L_2(\mathbb{R}^2)$. The perturbation level δ is assumed to be known. Previously, the authors investigated (accuracy estimates were obtained) global discrete regularizing algorithms for approximating the set of lines of discontinuity of a noisy function provided that the line of discontinuity of the exact function satisfies the local Lipschitz condition. In this paper, we introduce a one-sided Lipschitz condition and formulate a new, wider correctness class. New methods for localizing discontinuity lines are constructed that work on an extended class of functions. A convergence theorem is proved, and estimates of the approximation error and other important characteristics of the algorithms are obtained. It is shown that the new methods determine the position of the discontinuity lines with guarantee in situations where the standard methods do not work.

Keywords: ill-posed problems, regularization method, discontinuity line, global localization, discretization, Lipschitz condition.

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