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Quasilinear Equations with a Sectorial Set of Operators at Gerasimov–Caputo Derivatives

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Abstract—The issues of unique solvability of the Cauchy problem are studied for a quasilinear equation solved with respect to the highest fractional Gerasimov–Caputo derivative in a Banach space with closed operators from the class $A_{\alpha,G}^n$ in the linear part and with a nonlinear operator continuous in the graph norm. A theorem on the local existence and uniqueness of a solution to the Cauchy problem is proved in the case of a locally Lipschitz nonlinear operator. Under the nonlocal Lipschitz condition for the nonlinear operator, the existence of a unique solution on a predetermined interval is shown. Abstract results are illustrated by examples of initial–boundary value problems for partial differential equations with Gerasimov–Caputo time derivatives.

Keywords: Gerasimov–Caputo fractional derivative, Cauchy problem, sectorial set of operators, resolving family of operators, quasilinear equation, local solution, nonlocal solution, initial–boundary value problem.

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