ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2023, Vol. 321, Suppl. 1, pp. S216–S229.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2023, Vol. 29, No. 2, pp. 157–171.

On Essential Values of Oscillation Exponents for Solutions of a Linear Homogeneous Two-Dimensional Differential System

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Received February 27, 2023; revised April 17, 2023; accepted April 24, 2023

Abstract—In this paper, we study various types of oscillation exponents for solutions of linear homogeneous differential systems with continuous bounded coefficients. The calculation of the oscillation exponents is carried out by averaging the number of zeros (signs, roots, or hyperroots) of the projection of a solution x of a differential system onto any straight line, and this line is chosen so that the resulting average value is minimal: if the minimization is performed before (after) the averaging, then weak (strong, respectively) oscillation exponents are obtained. In the calculation of the oscillation exponents for a solution y of a linear homogeneous nth-order differential equation, a transition to the vector function $x = (y, \dot{y}, \ldots, y^{(n-1)})$ is carried out. In the first part of the paper, for any preassigned positive integer N, a two-dimensional periodic linear differential system is constructed, which has the property that its spectra of all upper and lower strong and weak oscillation exponents of strict and nonstrict signs, zeros, roots, and hyperroots contain the same set consisting of N different essential values, both metrically and topologically. Moreover, all these values are implemented on the same set of solutions of the constructed system; that is, for each solution from this set, all the oscillation exponents coincide with each other. In the second part of the paper, a similar theorem on the existence of a two-dimensional differential system with a countable set of essential (both metrically and topologically) values of oscillation exponents is proved. In constructing the mentioned systems and proving the required results, we use analytical methods of the qualitative theory of differential equations and methods of the theory of perturbations of solutions of linear differential systems, in particular, the author's technique for controlling the fundamental matrix of solutions of such systems in one special case.

Keywords: differential equation, linear system, oscillation, the number of zeros, oscillation exponents, Sergeev's frequency.

DOI: 10.1134/S0081543823030185

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