Richardson Method for a Diffusion Equation with Functional Delay

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Abstract—A diffusion equation with a functional delay effect is considered. The problem is discretized. Constructions of the Crank–Nicolson difference method with piecewise linear interpolation and extrapolation by continuation are given; the method here has the second order of smallness with respect to the sampling steps in time Δ and space h. The basic Crank–Nicolson method with piecewise cubic interpolation and extrapolation by continuation is constructed. The order of the residual without interpolation of the basic method is studied, and the expansion coefficients of the residual with respect to Δ and h are written. An equation for the leading term of the asymptotic expansion of the global error is written. Under certain assumptions, the validity of the application of the Richardson extrapolation procedure is substantiated and an appropriate method is constructed. The main of these assumptions is the consistency of the orders of smallness of Δ and h. It is proved that the method has order $O(\Delta^4 + h^4)$. The results of numerical experiments on test examples are presented.

Keywords: diffusion equation, functional delay, Crank–Nicolson method, piecewise cubic interpolation, extrapolation by continuation, Richardson method.

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