ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2023, Vol. 321, Suppl. 1, pp. S138–S153.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2023, Vol. 29, No. 2, pp. 87–103.

Assimilation of Boundary Data for Reconstructing the Absorption Coefficient in a Model of Stationary Reaction–Convection–Diffusion

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Received March 3, 2023; revised March 17, 2023; accepted March 20, 2023

Abstract—Direct and inverse problems for a model of stationary reaction–convection–diffusion are studied. The direct problem is to find a solution to the corresponding boundary value problem for given parameters of the model. Solvability conditions are specified for the direct problem, a priori estimates of the solution are presented, and the continuous dependence of the solution to the direct problem on a number of parameters is established. The inverse problem consists in finding the a priori unknown absorption coefficient of the medium, which characterizes the decay of some substance (or the heat sink) in a chemical process. The results of measuring the concentration of a substance (or its temperature) on an available part of the boundary of the domain filled with the corresponding medium (the domain of change of the spatial variable) are used as additional information for solving the inverse problem. It is proved that the inverse problem is ill-posed. Examples are given demonstrating that the inverse problem is unstable with respect to changes of the measured value and may have several solutions. To solve the inverse problem, a variational method based on the minimization of some appropriate residual functional (a target functional) is suggested. The extremal properties of the problem of minimizing the residual functional are studied. An explicit analytical formula is found for calculating the gradient of the residual functional, and the corresponding adjoint system and optimality system are written. Several stable iterative methods for minimizing the residual functional are specified. Results of numerical simulation of the solution to the inverse problem are presented.

Keywords: reaction–convection–diffusion equation, direct problem, inverse problem, residual functional, functional gradient, adjoint system, variational method, gradient minimization methods.

DOI: 10.1134/S0081543823030136

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