

# Operator Estimates in Two-Dimensional Problems with a Frequent Alternation in the Case of Small Parts with the Dirichlet Condition

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**Abstract**—A two-dimensional boundary value problem is studied for a general scalar elliptic second-order equation of the general form with frequent alternation of boundary conditions. The alternation is defined on small, closely spaced parts of the boundary on which the Dirichlet boundary condition and the nonlinear Robin boundary condition are set alternately. The distribution and size of these segments are arbitrary. The case is considered when, upon homogenization, the Dirichlet boundary condition completely disappears and only the original nonlinear Robin boundary condition remains. The main result is estimates for the  $W_2^1$ - and  $L_2$ -norms of the difference between the solutions of the perturbed and homogenized problems, which are uniform in the  $L_2$ -norm of the right-hand side and characterize the rate of convergence. It is shown that these estimates are order sharp.

**Keywords:** two-dimensional boundary value problem, elliptic equation, frequent alternation, homogenization, operator estimate.

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