ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2023, Vol. 321, Suppl. 1, pp. S4–S19.
© Pleiades Publishing, Ltd., 2023.
Russian Text © The Author(s), 2023,
published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2023, Vol. 29, No. 1, pp. 7–23.

Optimal Recovery on Classes of Functions Analytic in an Annulus

O. V. Akopyan[1,*] **and R. R. Akopyan**[2,**]

Received February 10, 2023; revised February 27, 2023; accepted February 27, 2023

Abstract—Let $C_{r,R}$ be an annulus with boundary circles γ_r and γ_R centered at zero; its inner and outer radii are r and R, respectively, $0 < r < R < \infty$. On the class of functions analytic in the annulus $C_{r,R}$ with finite L^2-norms of the angular limits on the circle γ_r and of the nth derivatives (of the functions themselves for $n = 0$) on the circle γ_R, we study interconnected extremal problems for the operator ψ_ρ^m that takes the boundary values of a function on γ_r to its restriction (for $m = 0$) or the restriction of its mth derivative (for $m > 0$) to an intermediate circle γ_ρ, $r < \rho < R$. The problem of the best approximation of ψ_ρ^m by bounded linear operators from $L^2(\gamma_r)$ to $C(\gamma_\rho)$ is solved. A method for the optimal recovery of the mth derivative on an intermediate circle γ_ρ from L^2-approximately given values of the function on the boundary circle γ_r is proposed and its error is found. The Hadamard–Kolmogorov exact inequality, which estimates the uniform norm of the mth derivative on an intermediate circle γ_ρ in terms of the L^2-norms of the limit boundary values of the function and the nth derivative on the circles γ_r and γ_R, is derived.

Keywords: analytic functions, Hadamard three-circle theorem, Kolmogorov's inequality, optimal recovery.

DOI: 10.1134/S0081543823030033

[1]Ural Federal University, Yekaterinburg, 620000 Russia
[2]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
e-mail: *olga_akopjan@rambler.ru, **RRAkopyan@mephi.ru