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On the Pringsheim Convergence of a Subsequence of Partial Sums of a Fourier Trigonometric Series

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Abstract—As follows from A. N. Kolmogorov's famous theorem (1925), partial sums of any integrable function f converge to f in measure. Therefore, if a subsequence of partial sums has a limit on a set of positive measure, then this subsequence can converge on this set to f only. Meantime, R. D. Getsadze (1986) showed that, in a space of dimension greater than 1 the cubic partial sums of an integrable function may diverge in measure. In the author's paper (1989), it was shown that one can choose a function in such a way that any subsequence of the cubic partial sums is unbounded almost everywhere. The following question remained open: is it true that if a subsequence of the cubic partial sums converges on a set of positive measure, then the limit coincides with the original function almost everywhere on this set? We give an affirmative answer to this question; moreover, not only for the cubic partial sums, but for Pringsheim's sums as well. The corresponding question for the spherical sums is still open. Subsequences of partial sums are connected with universal trigonometric series. We say that a d-dimensional trigonometric series is universal if, for any measurable d-dimensional function f that is 2π-periodic in each variable, there is a subsequence of partial sums of this series converging to f almost everywhere. This definition depends on the choice of the class of partial sums of the trigonometric series. As follows from M. G. Grigoryan's recent result (2022), for any d there is a d-dimensional trigonometric series that is universal both for Pringsheim's sums and for the spherical sums. Due to the main result of the present paper, a Fourier series cannot be universal for Pringsheim's sums.

Keywords: measurable functions, integrable functions, trigonometric Fourier series, Pringsheim convergence, subsequence of partial sums, almost everywhere convergence, Bernstein's summation method for Fourier series.

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