ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2022, Vol. 319, Suppl. 1, pp. S257–S270.
© Pleiades Publishing, Ltd., 2022.
Russian Text © The Author(s), 2022,
published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2022, Vol. 28, No. 3, pp. 188–201.

Volterra Functional Equations in the Theory of Optimization of Distributed Systems. On the Problem of Singularity of Controlled Initial–Boundary Value Problems

V. I. Sumin[1,2]

Received June 15, 2022; revised July 15, 2022; accepted July 18, 2022

Abstract—Earlier the author proposed a rather general form of describing controlled initial–boundary value problems (IBVPs) by means of Volterra functional equations (VFEs)

$$z(t) = f\left(t, A[z](t), v(t)\right), \quad t \equiv \{t^1, \dots, t^N\} \in \Pi \subset \mathbb{R}^N, \quad z \in L_p^m \equiv \left(L_p\left(\Pi\right)\right)^m,$$

where $f(\cdot,\cdot,\cdot) : \Pi \times \mathbb{R}^l \times \mathbb{R}^s \to \mathbb{R}^m$, $v(\cdot) \in \mathcal{D} \subset L_k^s$ is a control function, and $A : L_p^m\left(\Pi\right) \to L_q^l\left(\Pi\right)$ is a linear operator that is Volterra for some system $\mathbf{T}$ of subsets of Π in the following sense: for any $H \in \mathbf{T}$, the restriction $A\left[z\right]|_H$ does not depend on the values of $z|_{\Pi\setminus H}$, $p, q, k \in [1, +\infty]$. This definition of a Volterra operator is a multidimensional generalization of Tikhonov's known definition of a functional Volterra type operator. Various IBVPs for nonlinear evolution equations (hyperbolic, parabolic, integro-differential, with delays of different kinds, etc.) are naturally reduced (by inverting the main part) to such equations. The transition to an equivalent VFE-description of controlled IBVPs is adequate to many problems of distributed optimization (obtaining conditions for preserving the global solvability of IBVPs under perturbed controls, substantiation of numerical methods of optimal control, derivation of necessary optimality conditions, study of singular controls in necessary optimality conditions, etc.). In particular, based on such a description, the author proposed a scheme for deriving constructive sufficient conditions for the preservation (under a control perturbation) of the global solvability of controlled IBVPs. In the present paper, the effectiveness of the VFE-description of IBVPs for the theory of optimal control is demonstrated by an example of an IBVP for a controlled semilinear parabolic equation. The issues of obtaining sufficient conditions for the preservation (under a control perturbation) of the global solvability of IBVPs and deriving necessary optimality conditions for optimal control problems singular in the sense of J.-L. Lions are considered. It is shown that some optimization problems that were classified as singular can in fact be classified as nonsingular, since the necessary optimality conditions for them may be derived by bringing the problems to the classical form and varying the controls.

Keywords: Volterra functional equations, controlled initial–boundary value problems, conditions for preserving global solvability, singular optimality systems.

DOI: 10.1134/S0081543822060220

[1]Nizhny Novgorod State University, Nizhny Novgorod, 603950 Russia
[2]Tambov State University, Tambov, 392000 Russia
e-mail: v_sumin@mail.ru