

On the Baer–Suzuki Width of Some Radical Classes

J. Guo^{1,*}, W. Guo^{1,2,**}, D. O. Revin^{3,4,***}, and V. N. Tyutyanov^{5,****}

Received April 10, 2022; revised April 20, 2022; accepted April 25, 2022

Abstract—Let $\sigma = \{\sigma_i \mid i \in I\}$ be a fixed partition of the set of all primes into pairwise disjoint nonempty subsets σ_i . A finite group is called σ -nilpotent if it has a normal σ_i -Hall subgroup for any $i \in I$. Any finite group possesses a σ -nilpotent radical, which is the largest normal σ -nilpotent subgroup. In this note, it is proved that there exists an integer $m = m(\sigma)$ such that the σ -nilpotent radical of any finite group coincides with the set of elements x such that any m conjugates of x generate a σ -nilpotent subgroup. Other possible analogs of the classical Baer–Suzuki theorem are discussed.

Keywords: Baer–Suzuki width, σ -nilpotent group, σ -solvable group, complete class of groups.

DOI: 10.1134/S0081543822030075

¹Hainan University, Haikou, 570228 China

²University of Science and Technology of China, Hefei, 230026 China

³Sobolev Institute of Mathematics, Siberian Branch of the Russian Academy of Sciences, Novosibirsk, 630090 Russia

⁴Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia

⁵Gomel Branch of the International University “MITSO,” Gomel, 246029 Belarus

e-mail: *guojinecho@163.com, **wbguo@ustc.edu.cn, ***revin@math.nsc.ru, ****vtutanov@gmail.com