On the Weiss Conjecture. I

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Abstract—Let Γ be a connected finite graph, and let G be a vertex-transitive group of automorphisms of Γ such that the stabilizer G_x in G of a vertex x of Γ induces on the neighborhood $\Gamma(x)$ of x a primitive permutation group $G_x^{\Gamma(x)}$. The Weiss conjecture says that, under these assumptions, the order of G_x is bounded from above by a number depending only on the order $|\Gamma(x)|$ of Γ. In a research whose first part is the present paper, we show that some general results of the theory of finite groups can be used to provide a largely uniform analysis for a number of cases of the Weiss conjecture (including some cases that were not considered before). Although this first part is introductory, it makes possible to use certain previous results to confirm the Weiss conjecture for all primitive groups $G_x^{\Gamma(x)}$ different from groups of AS type and from groups of PA type (constructed on the basis of groups of AS type).

Keywords: graph, group of automorphisms, Weiss conjecture.

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