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Taylor Series for Resolvents of Operators on Graphs with Small Edges

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Abstract—We consider a second-order elliptic self-adjoint operator on a graph with small edges. Such a graph is obtained by shrinking a given graph by a factor of ε^{-1} and then gluing it to another fixed graph; here ε is a small positive parameter. No significant constraints are imposed on this pair of graphs. On such a graph, a general second-order self-adjoint elliptic operator is specified; its differential expression contains derivatives of all orders with variable coefficients and a variable potential. The boundary conditions at the vertices of the graph are also chosen in a general form. All coefficients both in the differential expression and in the boundary conditions can additionally depend on the small parameter ε; this dependence is assumed to be analytic. As established earlier, the parts of the resolvent of the operator corresponding to the restrictions of the resolvent to the edges of fixed length and to the small edges are analytic in ε as operators in the corresponding spaces, and the restriction to the small edges should be additionally sandwiched by a pair of dilatation operators. Analyticity means the possibility to represent these operators in the form of the corresponding Taylor series. The first main result of the paper is a procedure similar to the matching of asymptotic expansions for the recursive determination of all coefficients of these Taylor series. The second main result is the representation of the resolvent by a convergent series similar to a Taylor series with effective estimates of the remainders.

Keywords: graph, small edge, elliptic operator, resolvent, analyticity, Taylor series, matching of asymptotic expansions.

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