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On the $\mathfrak{F}$-Norm of a Finite Group

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Abstract—Let G be a finite group, and let $\mathfrak{F}$ be a nonempty formation. Then the intersection of the normalizers of the $\mathfrak{F}$-residuals of all subgroups of G is called the $\mathfrak{F}$-*norm* of G and is denoted by $N_{\mathfrak{F}}(G)$. A group G is called $\mathfrak{F}$-*critical* if $G \notin \mathfrak{F}$, but $U \in \mathfrak{F}$ for any proper subgroup U of G. We say that a finite group G is *generalized* $\mathfrak{F}$-*critical* if G contains a normal subgroup N such that $N \leq \Phi(G)$ and the quotient group G/N is $\mathfrak{F}$-critical. In this publication, we prove the following result: *If G does not belong to the nonempty hereditary formation $\mathfrak{F}$, then the $\mathfrak{F}$-norm $N_{\mathfrak{F}}(G)$ of G coincides with the intersection of the normalizers of the $\mathfrak{F}$-residuals of all generalized $\mathfrak{F}$-critical subgroups of G. In particular, the norm $N(G)$ of G coincides with the intersection of the normalizers of all cyclic subgroups of G of prime power order.*

Keywords: finite group, hereditary formation, $\mathfrak{F}$-residual of a group, $\mathfrak{F}$-norm of a group, generalized $\mathfrak{F}$-critical group.

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