ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2022, Vol. 317, Suppl. 1, pp. S121–S129.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2022, Vol. 28, No. 1, pp. 199–208.

Inverse Problems in the Class of Distance-Regular Graphs of Diameter 4

A. A. Makhnev[1,2,*] and D. V. Paduchikh[1,]**

Received October 14, 2021; revised January 19, 2022; accepted January 24, 2022

Abstract—For a distance-regular graph Γ of diameter 4, the graph $\Delta = \Gamma_{1,2}$ can be strongly regular. In this case, the graph $\Gamma_{3,4}$ is strongly regular and complementary to Δ. Finding the intersection array of Γ from the parameters of $\Gamma_{3,4}$ is an inverse problem. In the present paper, the inverse problem is solved in the case of an antipodal graph Γ of diameter 4. In this case, $r = 2$ and $\Gamma_{3,4}$ is a strongly regular graph without triangles. Further, Γ is an $AT4(p, q, r)$-graph only in the case $q = p + 2$ and $r = 2$. Earlier the authors proved that an $AT4(p, p+2, 2)$-graph does not exist. A Krein graph is a strongly regular graph without triangles for which the equality in the Krein bound is attained (equivalently, $q^2_{22} = 0$). A Krein graph $\mathrm{Kre}(r)$ with the second eigenvalue r has parameters $((r^2 + 3r)^2, r^3 + 3r^2 + r, 0, r^2 + r)$. For the graph $\mathrm{Kre}(r)$, the antineighborhood of a vertex is strongly regular with parameters $((r^2 + 2r - 1)(r^2 + 3r + 1), r^3 + 2r^2, 0, r^2)$ and the intersection of the antineighborhoods of two adjacent vertices is strongly regularly with parameters $((r^2 + 2r)(r^2 + 2r - 1), r^3 + r^2 - r, 0, r^2 - r)$. Let Γ be an antipodal graph of diameter 4, and let $\Delta = \Gamma_{3,4}$ be a strongly regular graph without triangles. In this paper it is proved that Δ cannot be a graph with parameters $((r^2 + 2r - 1)(r^2 + 3r + 1), r^3 + 2r^2, 0, r^2)$, and if Δ is a graph with parameters $((r^2 + 2r)(r^2 + 2r - 1), r^3 + r^2 - r, 0, r^2 - r)$, then $r > 3$. It is proved that a distance-regular graph with intersection array $\{32, 27, 12(r - 1)/r, 1; 1, 12/r, 27, 32\}$ exists only for $r = 3$, and, for a graph with array $\{96, 75, 32(r - 1)/r, 1; 1, 32/r, 75, 96\}$, we have $r = 2$.

Keywords: distance-regular graph, antipodal graph, graph Γ with strongly regular graph $\Gamma\ i, j$.

DOI: 10.1134/S0081543822030105

[1]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
[2]Ural Federal University, Yekaterinburg, 620000 Russia
e-mail: *makhnev@imm.uran.ru, **dpaduchikh@gmail.com