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Lotka–Volterra Competition Model with a Nonmonotone Therapy Function for Finding Optimal Strategies in the Treatment of Blood Cancers

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Abstract—The interaction of healthy and cancerous cell concentrations in diseases associated with blood cancer is described by a two-dimensional Lotka–Volterra competition model. A differential equation specifying the change in the concentration of a chemotherapeutic drug during treatment is added to the model. This equation includes a bounded control function determining the rate at which such a drug enters the patient's bloodstream. The effectiveness of the used treatment is described by a nonmonotone therapy function. The problem is to minimize the weighted difference between the concentrations of cancerous and healthy cells at the end of a given treatment period for the considered three-dimensional control system. Application of the Pontryagin maximum principle allows to analytically study the properties of the optimal control. We single out and investigate possible cases when such a control is a bang-bang function and also the cases when, along with the bang-bang sections, it can contain singular regimes of the first and second orders. The established analytical results are confirmed by numerical calculations performed for different values of parameters and initial conditions of the considered minimization problem.

Keywords: Lotka–Volterra competition model, nonmonotone therapy function, nonlinear control system, Pontryagin maximum principle, switching function, bang-bang control, singular regime, chattering.

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