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On a Quadratic Minimization Problem with Nonuniform Perturbations in the Criteria and Constraints

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Abstract—A quadratic minimization problem is considered in Hilbert spaces in the presence of a linear operator constraint of the equality type and a quadratic constraint of the inequality type. The problem is reformulated as the problem of finding the saddle point of the normal Lagrange function. For the numerical solution of this problem, a regularized gradient-type method is proposed that iterates both in primal and in dual variables. Approximate solutions are constructed under nonclassical information conditions, when the approximations to the exact operators from the problem statement available to the computer approximate these operators only strongly pointwise and there are no corresponding error estimates in the operator norms of the original spaces. Instead, a priori information is used about the error levels that become accessible when the norms are changed in the domains or ranges of the operators. Estimates of the first type appear after strengthening norms in the domain spaces of the operators, and estimates of the second type appear after weakening norms in their range spaces. At each iteration of the proposed method, two main actions are performed. First, the next approximation to the minimum value of the functional is computed using error estimates of the first type. Then, the next approximation to the optimal solution is found using error estimates of the second type. It is proved that the approximations generated by the proposed method converge to the solution of the original optimization problem in the norm of the original space.

Keywords: quadratic minimization problem, approximate data, numerical solution, ill-posed problem, regularized gradient method, Lagrange function, saddle point.

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