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Analog of the Hadamard Theorem and Related Extremal Problems on the Class of Analytic Functions

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Abstract—We study several related extremal problems for analytic functions in a finitely connected domain G with rectifiable Jordan boundary Γ. A sharp inequality is established between values of a function analytic in G and weighted means of its boundary values on two measurable subsets γ_1 and $\gamma_0 = \Gamma \setminus \gamma_1$ of the boundary:

$$|f(z_0)| \le \mathcal{C}\, \|f\|^{\alpha}_{L^q_{\varphi_1}(\gamma_1)}\, \|f\|^{\beta}_{L^p_{\varphi_0}(\gamma_0)}, \quad z_0 \in G, \quad 0 < q, p \le \infty.$$

The inequality is an analog of Hadamard's three-circle theorem and the Nevanlinna brothers' two-constant theorem. In the case of a doubly connected domain G and $1 \le q, p \le \infty$, we study the cases where the inequality provides the value of the modulus of continuity for a functional of analytic extension of a function from the part γ_1 of the boundary to a given point of the domain. In these cases, the corresponding problem of optimal recovery of a function from its approximate boundary values on γ_1 and the problem of the best approximation of a functional by bounded linear functionals are solved. The case of a simply connected domain G has been completely investigated previously.

Keywords: analytic functions, optimal recovery of a functional, best approximation of an unbounded functional by bounded functionals, harmonic measure.

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