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On Stable Reconstruction of Analytic Functions from Fourier Samples

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Abstract—Stability of reconstruction of analytic functions from the values of $2m+1$ coefficients of its Fourier series is studied. The coefficients can be taken from an arbitrary symmetric set $\delta_m \subset \mathbb{Z}$ of cardinality $2m+1$. It is known that, for $\delta_m = \{j : |j| \le m\}$, i.e., if the coefficients are consecutive, the fastest possible convergence rate in the case of stable reconstruction is an exponential function of the square root of m. Any method with faster convergence is highly unstable. In particular, exponential convergence implies exponential ill-conditioning. In this paper we show that if the sets (δ_m) are chosen freely, there exist reconstruction operators (ϕ_{δ_m}) that have exponential convergence rate and are almost stable; specifically, their condition numbers grow at most linearly: $\kappa_{\delta_m} < c\,m$. We also show that this result cannot be noticeably strengthened. More precisely, for any sets (δ_m) and any reconstruction operators (ϕ_{δ_m}), exponential convergence is possible only if $\kappa_{\delta_m} \ge c\,m^{1/2}$.

Keywords: Fourier coefficients, stable reconstruction, polynomial inequalities.

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