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Automorphisms of a Distance-Regular Graph with Intersection Array $\{30, 22, 9; 1, 3, 20\}$

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Abstract—A distance-regular graph Γ of diameter 3 is called a Shilla graph if it has the second eigenvalue $\theta_1 = a_3$. In this case $a = a_3$ divides k and we set $b = b(\Gamma) = k/a$. Koolen and Park obtained the list of intersection arrays for Shilla graphs with $b = 3$. There exist graphs with intersection arrays $\{12, 10, 5; 1, 1, 8\}$ and $\{12, 10, 3; 1, 3, 8\}$. The nonexistence of graphs with intersection arrays $\{12, 10, 2; 1, 2, 8\}$, $\{27, 20, 10; 1, 2, 18\}$, $\{42, 30, 12; 1, 6, 28\}$, and $\{105, 72, 24; 1, 12, 70\}$ was proved earlier. In this paper, we study the automorphisms of a distance-regular graph Γ with intersection array $\{30, 22, 9; 1, 3, 20\}$, which is a Shilla graph with $b = 3$. Assume that a is a vertex of Γ, $G = \mathrm{Aut}(\Gamma)$ is a nonsolvable group, $\bar{G} = G/S(G)$, and $\bar{T}$ is the socle of $\bar{G}$. Then $\bar{T} \cong L_2(7)$, A_7, A_8, or $U_3(5)$. If Γ is arc-transitive, then T is an extension of an irreducible $F_2U_3(5)$-module V by $U_3(5)$ and the dimension of V over F_3 is 20, 28, 56, 104, or 288.

Keywords: Shilla graph, graph automorphism.

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