On the Pronormality of Second Maximal Subgroups in Finite Groups with Socle $L_2(q)$

V. I. Zenkov[1,2]

Abstract—According to P. Hall, a subgroup H of a finite group G is called pronormal in G if, for any element g of G, the subgroups H and H^g are conjugate in $\langle H, H^g \rangle$. The simplest examples of pronormal subgroups of finite groups are normal subgroups, maximal subgroups, and Sylow subgroups. Pronormal subgroups of finite groups were studied by a number of authors. For example, Legovini (1981) studied finite groups in which every subgroup is subnormal or pronormal. Later, Li and Zhang (2013) described the structure of a finite group G in which, for a second maximal subgroup H, its index in $\langle H, H^g \rangle$ does not contain squares for any g from G. A number of papers by Kondrat'ev, Maslova, Revin, and Vdovin (2012–2019) are devoted to studying the pronormality of subgroups in a finite simple nonabelian group and, in particular, the existence of a nonpronormal subgroup of odd index in a finite simple nonabelian group. In The Kourovka Notebook, the author formulated Question 19.109 on the equivalence in a finite simple nonabelian group of the condition of pronormality of its second maximal subgroups and the condition of Hallness of its maximal subgroups. Tyutyanov gave a counterexample $L_2(2^{11})$ to this question. In the present paper, we provide necessary and sufficient conditions for the pronormality of second maximal subgroups in the group $L_2(q)$. In addition, for $q \leq 11$, we find the finite almost simple groups with socle $L_2(q)$ in which all second maximal subgroups are pronormal.

Keywords: finite group, simple group, maximal subgroup, pronormal subgroup.

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[1]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
[2]Ural Federal University, Yekaterinburg, 620000 Russia
e-mail: v1i9z52@mail.ru