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Asymptotics of a Solution to a Singularly Perturbed Time-Optimal Control Problem of Transferring an Object to a Set

A. R. Danilin[1,*] **and O. O. Kovrizhnykh**[1,2,**]

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Abstract—The present work is devoted to a time-optimal control problem for a singularly perturbed linear autonomous system with smooth geometric constraints on the control and an unbounded target set:

$$\begin{cases} \dot{x} = A_{11}x + A_{12}y + B_1u, & x \in \mathbb{R}^n,\ y \in \mathbb{R}^m,\ u \in \mathbb{R}^r, \\ \varepsilon\dot{y} = A_{21}x + A_{22}y + B_2u, & \|u\| \le 1, \\ x(0) = x_0 \neq 0, \quad y(0) = y_0, & 0 < \varepsilon \ll 1, \\ x(T_\varepsilon) = 0, \quad y(T_\varepsilon) \in \mathbb{R}^m, \quad T_\varepsilon \longrightarrow \min. \end{cases}$$

The uniqueness of the representation of the optimal control with a normalized defining vector in the limit problem is proved. The solvability of the problem is established. The limit relations for the optimal time and the vector determining the optimal control are obtained. An asymptotic analog of the implicit function theorem is proved and used to derive a complete asymptotics of the solution to the problem in powers of the small parameter ε.

Keywords: optimal control, time-optimal control problem, asymptotic expansion, singularly perturbed problem, small parameter.

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[1]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
[2]Ural Federal University, Yekaterinburg, 620000 Russia
e-mail: *dar@imm.uran.ru, **koo@imm.uran.ru