Optimal Strategies in the Treatment of Cancers in the Lotka–Volterra Mathematical Model of Competition

N. L. Grigorenko[1,*], E. N. Khailov[1,**], E. V. Grigorieva[2,***], and A. D. Klimenkova[1,****]

Abstract—The Lotka–Volterra competition model is applied to describe the interaction between the concentrations of healthy and cancerous cells in diseases associated with blood cancer. The model is supplemented with a differential equation characterizing the change in the concentration of a chemotherapeutic drug. The equation contains a scalar bounded control that specifies the rate of drug intake. We consider the problem of minimizing the weighted difference between the concentrations of cancerous and healthy cells at the end time of the treatment period. The Pontryagin maximum principle is used to establish analytically the properties of an optimal control. We describe situations in which the optimal control is a bang–bang function and situations in which the control may contain a singular arc in addition to bang–bang arcs. The results obtained are confirmed by corresponding numerical calculations.

Keywords: Lotka–Volterra competition model, nonlinear control system, Pontryagin maximum principle, switching function, bang–bang control, singular arc.

DOI: 10.1134/S0081543821030111

[1]Faculty of Computational Mathematics and Cybernetics, Moscow State University, Moscow, 119991 Russia
[2]Texas Woman's University, Denton, TX 76204, USA
e-mail: *grigor@cs.msu.su, **khailov@cs.msu.su, ***egrigorieva@mail.twu.edu, ****klimenkovaad@mail.ru