ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2021, Vol. 313, Suppl. 1, pp. S83–S99.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2020, Vol. 26, No. 1, pp. 51–70.

Step-Affine Functions, Halfspaces, and Separation of Convex Sets with Applications to Convex Optimization Problems

V. V. Gorokhovik[1]

Received November 11, 2019; revised January 10, 2020; accepted January 14, 2020

Abstract—We introduce the class of step-affine functions defined on a real vector space and establish the duality between step-affine functions and halfspaces, i.e., convex sets whose complements are convex as well. Using this duality, we prove that two convex sets are disjoint if and only if they are separated by some step-affine function. This criterion is actually the analytic version of the Kakutani–Tukey criterion of the separation of disjoint convex sets by halfspaces. As applications of these results, we derive a minimality criterion for solutions of convex vector optimization problems considered in real vector spaces without topology and an optimality criterion for admissible points in classical convex programming problems not satisfying the Slater regularity condition.

Keywords: step-affine functions, halfspaces, separation of convex sets, convex vector optimization problems, convex programming.

DOI: 10.1134/S008154382103010X

[1]Institute of Mathematics, National Academy of Sciences of Belarus, Minsk, 220072 Belarus
e-mail: gorokh@im.bas-net.by