ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2021, Vol. 313, Suppl. 1, pp. S14–S20.
© Pleiades Publishing, Ltd., 2021.
Russian Text © The Author(s), 2019,
published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2019, Vol. 25, No. 4, pp. 44–51.

Inverse Problems in the Theory of Distance-Regular Graphs: Dual 2-Designs

I. N. Belousov[1,2,*] and A. A. Makhnev[1,2,]**

Received August 1, 2019; revised November 8, 2019; accepted November 25, 2019

Abstract—Let Γ be a distance-regular graph of diameter 3 with a strongly regular graph Γ_3. Finding the parameters of Γ_3 from the intersection array of Γ is a direct problem, and finding the intersection array of Γ from the parameters of Γ_3 is its inverse. The direct and inverse problems were solved by A.A. Makhnev and M.S. Nirova: if a graph Γ with intersection array $\{k, b_1, b_2; 1, c_2, c_3\}$ has eigenvalue $\theta_2 = -1$, then the graph complementary to Γ_3 is pseudo-geometric for $pG_{c_3}(k, b_1/c_2)$. Conversely, if Γ_3 is a pseudo-geometric graph for $pG_\alpha(k,t)$, then Γ has intersection array $\{k, c_2t, k-\alpha+1; 1, c_2, \alpha\}$, where $k-\alpha+1 \le c_2t < k$ and $1 \le c_2 \le \alpha$. Distance-regular graphs Γ of diameter 3 such that the graph Γ_3 ($\bar\Gamma_3$) is pseudogeometric for a net or a generalized quadrangle were studied earlier. In this paper, we study intersection arrays of distance-regular graphs Γ of diameter 3 such that the graph Γ_3 ($\bar\Gamma_3$) is pseudogeometric for a dual 2-design $pG_{t+1}(l,t)$. New infinite families of feasible intersection arrays are found: $\{m(m^2-1), m^2(m-1), m^2; 1, 1, (m^2-1)(m-1)\}$, $\{m(m+1), (m+2)(m-1), m+2; 1, 1, m^2-1\}$, and $\{2m(m-1), (2m-1)(m-1), 2m-1; 1, 1, 2(m-1)^2\}$, where $m \equiv \pm 1 \pmod 3$. The known families of Steiner 2-designs are unitals, designs corresponding to projective planes of even order containing a hyperoval, designs of points and lines of projective spaces $PG(n,q)$, and designs of points and lines of affine spaces $AG(n,q)$. We find feasible intersection arrays of a distance-regular graph Γ of diameter 3 such that the graph Γ_3 ($\bar\Gamma_3$) is pseudogeometric for one of the known Steiner 2-designs.

Keywords: distance-regular graph, dual 2-design.

DOI: 10.1134/S0081543821030032

[1]Krasovskii Institute of Mathematics and Mechanics, Ural Branch of the Russian Academy of Sciences, Yekaterinburg, 620108 Russia
[2]Ural Federal University, Yekaterinburg, 620000 Russia
e-mail: [*]i_belousov@mail.ru, [**]makhnev@imm.uran.ru