ISSN 0081-5438, Proceedings of the Steklov Institute of Mathematics, 2021, Vol. 313, Suppl. 1, pp. S185–S193.

published in Trudy Instituta Matematiki i Mekhaniki UrO RAN, 2019, Vol. 25, No. 4, pp. 201–209.

On Periodic Groups with a Regular Automorphism of Order 4

A. I. Sozutov[1]

Received July 13, 2019; revised September 30, 2019; accepted October 21, 2019

Abstract—We study periodic groups of the form $G = F \leftthreetimes \langle a \rangle$ with the conditions $C_F(a) = 1$ and $|a| = 4$. The mapping $a : F \to F$ defined by the rule $t \to t^a = a^{-1}ta$ is a fixed-point-free (regular) automorphism of the group F. In this case, a finite group F is solvable and its commutator subgroup is nilpotent (Gorenstein and Herstein, 1961), and a locally finite group F is solvable and its second commutator subgroup is contained in the center $Z(F)$ (Kovács, 1961). It is unknown whether a periodic group F is always locally finite (Shumyatsky's Question 12.100 from The Kourovka Notebook). We establish the following properties of groups. For $\pi = \pi(F) \setminus \pi(C_F(a^2))$, the group F is π-closed and the subgroup $O_\pi(F)$ is abelian and is contained in $Z([a^2, F])$ (Theorem 1). A group F without infinite elementary abelian a^2-admissible subgroups is locally finite (Theorem 2). In a nonlocally finite group F, there is a nonlocally finite a-admissible subgroup factorizable by two locally finite a-admissible subgroups (Theorem 3). For any positive integer n divisible by an odd prime, we give examples of nonlocally finite periodic groups with a regular automorphism of order n.

Keywords: periodic group, regular (fixed-point-free) automorphism, solvability, local finiteness, nilpotency.

DOI: 10.1134/S0081543821030196

[1]Siberian Federal University, Krasnoyarsk, 660041 Russia
e-mail: sozutov_ai@mail.ru